RF & Microwave Engineering 101

SOLUTIONS

For the exercises of meeting 4:

Noise Figure measurements And nonlinearity

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RF & Microwave Engineering 101,

Solutions for Homework #4

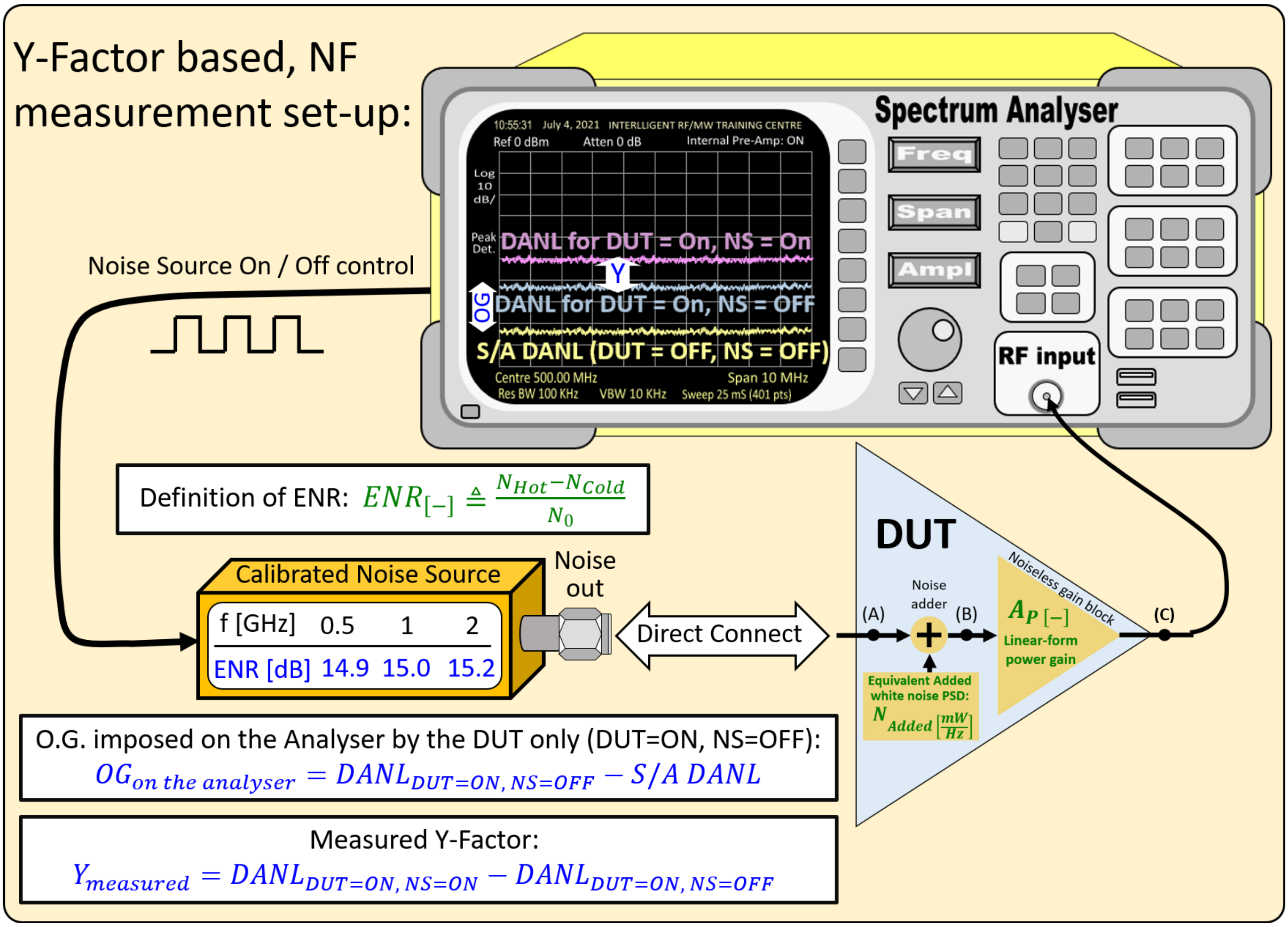
Homework 4 answers summary

Exercise number	Subject	Select your answer 1	2	3	4	5	Remarks
4.1A	Calculating the DUT's output noise PSD for the "DUT = ON, Noise Source = OFF" state		X				
4.1B	Calculating the DUT's output noise PSD for the "DUT = ON, Noise Source = ON" state				X		
4.1C	Calculating the DUT's output Y-Factor			X			
4.1D	Calculating the DUT's Noise factor / Noise Figure					X	
4.2	Even-odd decomposition	X	X	X	X	X	Please scan your answer
4.3	Memoryless nonlinearity, 3rd order odd "baseline" model	X	X	X	X	X	Please scan your answer

Exercise 4.1

Subject: Proving the Y-factor to Noise-factor relationship:

The following "Y-factor" based measurement set-up is used, in order to measure the Noise-Figure of an amplifier. The set-up includes a calibrated on-off switched Noise Source (NS) with a given ENR specification, a Device Under Test (DUT) and a Spectrum Analyser (SA):



The Calibrated Noise source serves as the calibration standard of the measurement. The Noise source includes its ENR versus frequency specification. ENR (Excessive Noise Ratio) is defined by the following (general case) definition: $ENR_{[-]} \triangleq \frac{N_{Hot}-N_{Cold}}{N_0}$. In most practical cases (including in our case), it is assumed that the "cold" PSD at the noise source's output is simply N_0.

This is a reasonable assumption since most (if not all) commercially available noise sources include an internal attenuator (connected between the noise generating diode to the device output), to improve the noise source's output port's Return Loss / matching into a pre-defined system-impedance (such as 50Ω). This attenuator is made of resistors which generate thermal noise at ambient temperature.

Therefore, for ambient temperatures close to the standard room temperature (T_0), a more practical ENR term can be used:

$$ENR_{\,for\ most\ practical\ noise\ sources\ (where\ N_{Cold}=N_0)} = \frac{N_{Hot} - N_0}{N_0}$$

4.1A: <u>Calculating the DUT's output noise PSD for the "DUT = ON, Noise Source = OFF" state:</u>

Assuming the practical case where the "Cold" noise output of the Noise Source is indeed $N_{Cold} = N_0$, formulate the amplifier's (DUT) output noise PSD, for the case when the Noise Source is switched OFF and the amplifier (DUT) is ON. Mark the correct result:

<u>The correct solution is:</u>

4.1A2: The DUT's output Noise PSD (for: DUT=ON, NS=OFF) is: $N_{(c)} = N_0 \cdot A_{p\ DUT} \cdot F_{DUT}$

<u>Explanation:</u>

Because the amplifier "sees" an input noise PSD of N_0, we can use the output noise PSD formula of a "receiving" system, which is exactly the requested term.

4.1B: <u>Calculating the DUT's output noise PSD for the "DUT = ON, Noise Source = ON" state:</u>

Now, formulate the amplifier's (DUT) output noise PSD, for the case when the Noise Source is switched ON and the amplifier (DUT) is also ON. Mark the correct result:

<u>The correct solution is:</u>

4.1B4: The DUT's output Noise PSD (for: DUT=ON, NS=ON) is: $N_{(c)} = N_0 \cdot A_{p\ DUT} \cdot (ENR_{NS} + F_{DUT})$

<u>Explanation:</u>

Let's begin by formulating the input noise PSD of the noise source, when it is switched on. Since the noise source provides room-temperature noise when switched OFF, we can use the practical ENR term:

$$ENR_{\,for\ most\ practical\ noise\ sources\ (where\ N_{Cold}=N_0)} = \frac{N_{Hot} - N_0}{N_0}$$

Which yields a "hot" ("on" state) noise PSD at the noise source's output of:

$$N_{NS,\ \ Hot\ (on)} = (ENR_{NS} + 1)N_0$$

Now, according to the IEEE's noisy amplifier model, the added noise of the amplifier is given by (regardless of the input noise PSD to the amplifier):

$$N_{added,\ \ DUT} = (F_{DUT} - 1)N_0$$

Therefore, when both the noise source, as well as the amplifier are "On", the noise PSD at point "B" within the amplifier's noise equivalent model is.

$$N_{(B)} = N_{NS,\ \ Hot} + N_{added,\ \ DUT} = (ENR_{NS} + 1 + F_{DUT} - 1)N_0 = (ENR_{NS} + F_{DUT})N_0$$

Finally, to obtain the device's output noise PSD, we need to multiply the noise PSD at point "B" by the linear power amplification, $A_{p\ DUT}$, which yields our requested result:

The DUT's output Noise PSD (for: DUT=ON, NS=ON) is: $N_{(c)} = N_0 \cdot A_{p\ DUT} \cdot (ENR_{NS} + F_{DUT})$

4.1C: Calculating the DUT's output Y-Factor:

The DUT's output Y-factor is defined by $Y_{DUT\ out\ (c)} \triangleq \frac{N_{(c)\ where\ DUT=ON,\ \ NS=ON}}{N_{(c)\ where\ DUT=ON,\ \ NS=OFF}}$.

It represents the ratio between:

- The output noise PSD of the DUT, while DUT = ON, NS = ON
  and:
- The output noise PSD of the DUT, while DUT = ON, NS = OFF

Based upon your two results for the DUT's output noise PSD at the 2 NS states, calculate the DUT's output Y-factor. Mark the correct result:

The correct solution is:

4.1C3: The DUT's output Y-factor is: $Y_{(c)} = 1 + \frac{ENR_{NS}}{F_{DUT}}$

Explanation:

This term is simply obtained by dividing the 2 results of 4.1B by 4.1A.

4.1D: Calculating the DUT's Noise factor / Noise Figure:

Based on your previous calculation, formulate the DUT's Noise Factor (or Noise Figure), using the given ENR and the (measured) Y-Factor:

The correct solution is:

4.1D5: The DUT's Noise Factor is: $F_{DUT} = \frac{ENR_{NS}}{Y_{(c)}-1}$

Explanation:

This term is simply obtained re-formulating the 4.1C result.

is: $F_{DUT} = \frac{ENR_{NS}}{Y_{(c)}-1}$

Exercise 4.2

Subject: Subject: Even-odd decomposition

Decompose the following "general" function $m(x)$ where $x \in \mathbb{R}$, $m(x) \in \mathbb{R}$) into a sum of an even function, $f(x)$ and an odd function, $g(x)$, where: $m(x) = f(x) + g(x)$:

$$m(x) = 4x^4 + (2x-3)^3 + 5$$

The correct solution is:

$$m(x) = 4x^4 + (8x^3 - 36x^2 + 54x - 27) + 5 = 4x^4 + 8x^3 - 36x^2 + 54x - 22$$

Therefore:

$$f(x) = m_{even}(x) = \frac{m(x) + m(-x)}{2} = x^4 - 36x^2 - 22$$

$$g(x) = m_{odd}(x) = \frac{m(x) - m(-x)}{2} = 8x^3 + 54x$$

Exercise 4.3

Subject: Memoryless nonlinearity, 3rd order odd "baseline" model:

A two-tone input test signal, $V_{in}(t) = A\cos(\omega_1 t) + B\cos(\omega_2 t)$ is inserted into a nonlinear device, with an odd input / output voltage transfer function, given by $V_{out}(V_{in}) \cong a_1[V_{in}(t)] + a_3[V_{in}(t)]^3$. This odd model 3rd order will serve us in our presentation as our "baseline" model for memoryless nonlinearity analysis. Assuming the nonlinear device is represented by the above "baseline" model, and given the input 2-tone signal, you are requested to develop the full expression for the voltage output of the device.
After you will obtain that full expression, you are requested to sort its components according to their frequency, and to fill-in the right-hand columns in the table below. Line 1 is already filled as an example.

Solution (based on slide 4.34, with an extra "bonus" column of the log-amplitude dependency):

#	Output frequency	Exact voltage expression for the output at this frequency, according to the 3rd order odd "baseline mathematical model" selected	Small signal approximation for the voltage expression (most dominant term)	Log-Amplitude dependency (small sig.) $1dB_{in} \to xdB_{out}$
1	ω_1	$\left[a_1 A + a_3\left(\frac{3A^3}{4} + \frac{3B^2 A}{2}\right)\right]\cos(\omega_1 t)$	$a_1 A\cos(\omega_1 t)$	$A: 1_{dB} \to 1_{dB}$
2	ω_2	$\left[a_1 B + a_3\left(\frac{3B^3}{4} + \frac{3A^2 B}{2}\right)\right]\cos(\omega_2 t)$	$a_1 B\cos(\omega_2 t)$	$B: 1_{dB} \to 1_{dB}$
3	$3\omega_1$	$\frac{a_3 A^3}{4}\cos(3\omega_1 t)$	$\frac{a_3 A^3}{4}\cos(3\omega_1 t)$	$A: 1_{dB} \to 3_{dB}$
4	$3\omega_2$	$\frac{a_3 B^3}{4}\cos(3\omega_2 t)$	$\frac{a_3 B^3}{4}\cos(3\omega_2 t)$	$B: 1_{dB} \to 3_{dB}$
5	$2\omega_1 + \omega_2$	$\frac{3\,a_3 A^2 B}{4}\cos[(2\omega_1 + \omega_2)t]$	$\frac{3\,a_3 A^2 B}{4}\cos[(2\omega_1 + \omega_2)t]$	$A: 1_{dB} \to 2_{dB}$ $B: 1_{dB} \to 1_{dB}$
6	$2\omega_2 + \omega_1$	$\frac{3\,a_3 B^2 A}{4}\cos[(2\omega_2 + \omega_1)t]$	$\frac{3\,a_3 B^2 A}{4}\cos[(2\omega_2 + \omega_1)t]$	$A: 1_{dB} \to 1_{dB}$ $B: 1_{dB} \to 2_{dB}$
7	$2\omega_1 - \omega_2$	$\frac{3\,a_3 A^2 B}{4}\cos[(2\omega_1 - \omega_2)t]$	$\frac{3\,a_3 A^2 B}{4}\cos[(2\omega_1 - \omega_2)t]$	$A: 1_{dB} \to 2_{dB}$ $B: 1_{dB} \to 1_{dB}$
8	$2\omega_2 - \omega_1$	$\frac{3\,a_3 B^2 A}{4}\cos[(2\omega_2 - \omega_1)t]$	$\frac{3\,a_3 B^2 A}{4}\cos[(2\omega_2 - \omega_1)t]$	$A: 1_{dB} \to 1_{dB}$ $B: 1_{dB} \to 2_{dB}$