INTERLLIGENT

RF & Microwave Solutions

RF & Microwave Engineering 101

Presentation for meeting 4 of 20:

Introduction to Nonlinear device characterization (part 1 of 2)

By Oren Hagai

Meeting 4, April 6th, 2021

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Content for meeting 4:

Introduction to nonlinear device characterization

1. Introduction: Basic terms and motivation for this subject

2. A few math reminders before we begin dealing with RF nonlinearities:

2.1: Definitions of Even and Odd function
2.2: Decomposition of “general” functions into even and odd parts
2.3: McLaurin Series representation of a function
2.4: McLaurin Series private cases for even / odd functions
2.5: Small signal approximation
2.6: The spectral decomposition of the terms $cos^n(\omega t)$, $sin^n(\omega t)$

3. Memoryless RF nonlinearity analysis:

3.1: 3rd order approximation representation for “base model” for analysis
3.2: Two-tone analysis

Content for meeting 4 (continued):

Introduction to nonlinear device characterization

3. Memoryless RF nonlinearity analysis (contunued):

3.3: Results summary

4. Interception points:

4.1: The concept of interception points

4.2: Real life results generalization

5. Measuring nonlinear products by a spectrum analyser

6. Last homework review + NF ADS demo

Nonlinear device characterization, part 1:
Introduction, basic terms and motivation

Introduction:

AM-AM and AM-PM nonlinearities in RF and microwave devices is a fundamental subject in microwave engineering.

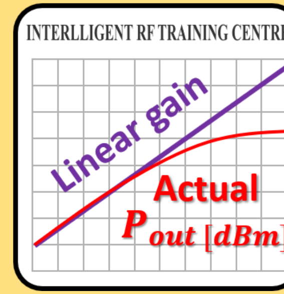
- **The device's nonlinear behavior has monumental influence on the overall RF system performance.**
- **Nonlinear behavior causes new frequency products (harmonics and intermodulation distortion), spectral regrowth and deteriorate the wanted signal's quality (in terms of EVM or other modulation quality metrics).**

Effects of memoryless nonlinearity:

(1) Gain compression:

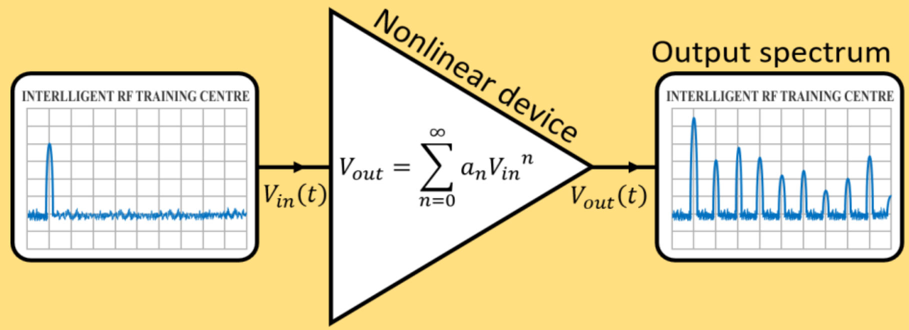
Deviation from "small signal" (nominal) linear gain at high power levels, due to (higher than 1st order) nonlinear effects (a frequency selective measurement) which become more and more dominant at higher power levels.

Power sweep



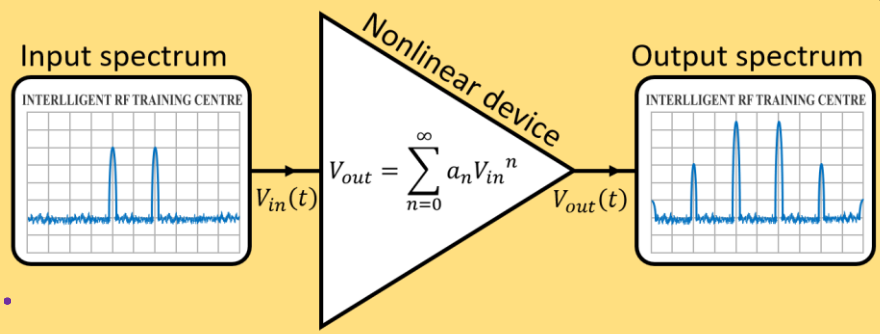
(2) Harmonics:

Integer multiples of an input's signal frequency, produced by the nonlinear device. Requires a minimum of one CW input to exist.



(3) Intermodulation distortion:

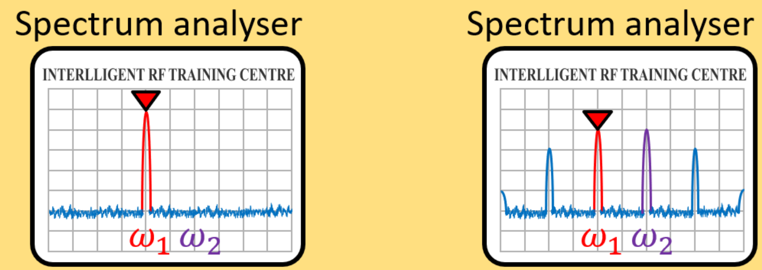
New spectral products generated by the nonlinear device at integer linear combinations of the input frequencies, Requires a minimum of 2 input tones to exist.



Effects of memoryless nonlinearity:

(4) Blocking (also known as de-sensing) effects:

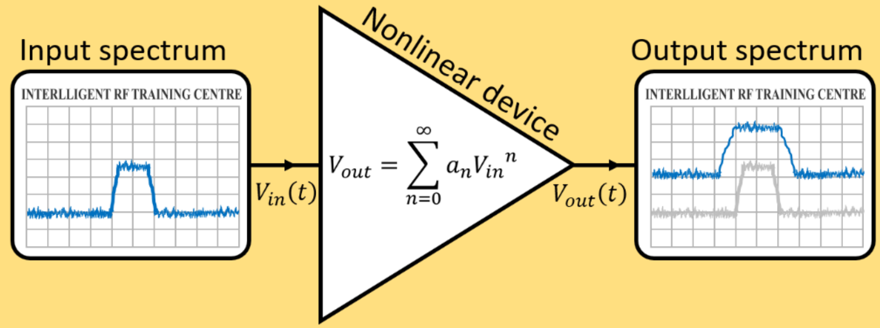
The output power of a carrier will decrease as a second input carrier is inserted to the nonlinear device.



Single CW input at ω_1 Dual CW input at ω_1 and ω_2

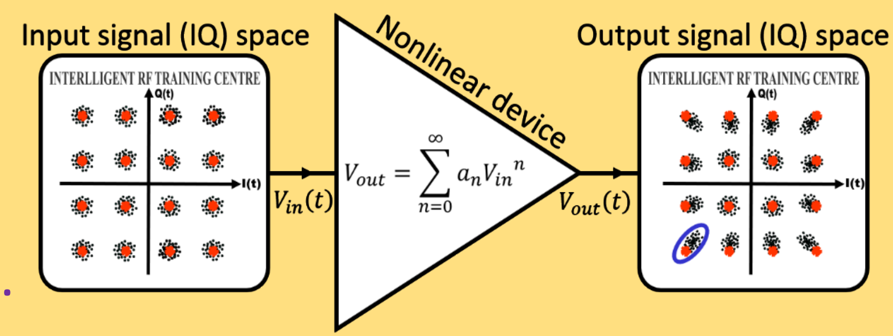
(5) Spectral regrowth:

An expansion of the wideband (modulated) signal's spectral-emission mask due to the nonlinear effects of the device which the signal is passing through.



(6) Modulation distortion:

When a digitally modulated constellation is inserted into a nonlinear device, its higher-power (exterior) symbols will suffer more (than the low power, internal symbils) from gain compression and will be displaced towards the centre of origin.



Kinds of nonlinearity

Memoryless nonlinearity (AM-AM)

Nonlinearity with memory

The input-output relation between V_{in} and V_{out} is time invariant and is given by a McLaurin power series.

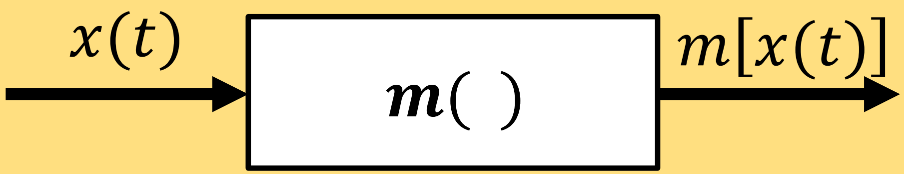
In other words, the input amplitude controls the output amplitude.

The input-output relation between V_{in} and V_{out} is time variant and is given by a Volterra power series.

In other words, the input amplitude dominates the output phase.

Basic systems definitions:

Definition of system Linearity: *Systems that satisfy the principle of superposition (which includes both homogeneity and additivity) are refed as* ***linear systems.***

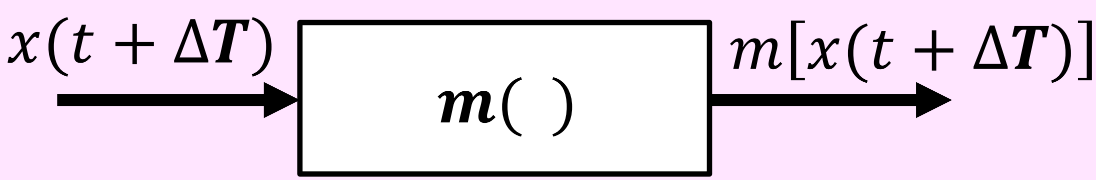


Conditions for linearity (collectively called "superposition"):

(1) Additivity: $m[x_1(t) + x_2(t)] = m[x_1(t)] + m[x_2(t)]$

(2) Homogeneity: $m[\alpha x(t)] = \alpha m[x(t)]$ where α is any complex constant

Definition of system's time invariance: *A system is called* ***time-invariant*** *if a time-shift in its input signal results an identical time-shift in the output signal*:



LTI (Linear Time Invariant) systems satisfy linearity and time-invariance conditions together. It can be shown that LTI systems cannot generate new frequency products.

Nonlinear device characterization, part 2:

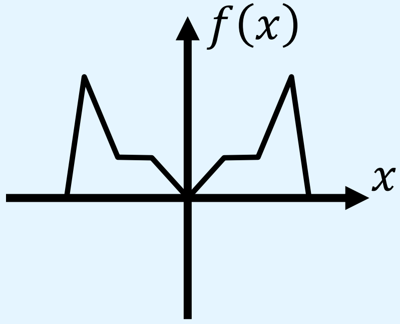
A few math reminders before we begin dealing with RF nonlinearities

2.1 Even and odd functions:

Definition of a real-valued EVEN function:
Let f be a real-valued function of a real variable. Then f is **even** if the following equation holds for all x such that x and $-x$ in are in the domain of f: $\boldsymbol{f(x) = f(-x)}$**.** In other words, **there is symmetry about the y-axis** (like a reflection).

A few examples for EVEN functions:
$\cos(x), \cosh(x), x^N \ (for\ even\ integer\ N\ values), |x|$

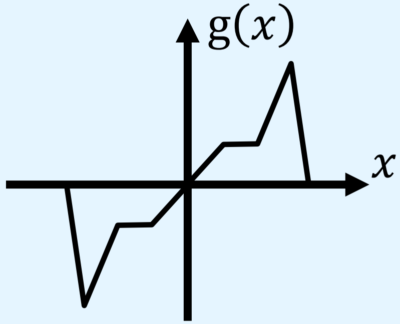
Example illustration for an arbitrary even function:



Definition of a real-valued ODD function:
Let g be a real-valued function of a real variable. Then g is **odd** if the following equation holds for all x such that x and $-x$ in are in the domain of g: $\boldsymbol{g(x) = -g(-x)}$. In other words, **there is symmetry about the origin**.

A few examples for ODD functions:
$\sin(x), \sinh(x), x^N \ (for\ odd\ integer\ N\ values), \mathrm{erf}(x)$

Example illustration for an arbitrary odd function:



2.2: Decomposition of "general" functions into even and odd parts:

Even-Odd function decomposition:

A "general" real-valued function, $m(x)$, of a real variable x, can be decomposed into a sum of its even and odd parts (functions):

$$\underbrace{m(x)}_{\text{"general" function}} = \underbrace{f(x)}_{\text{even part}} + \underbrace{g(x)}_{\text{odd part}}$$

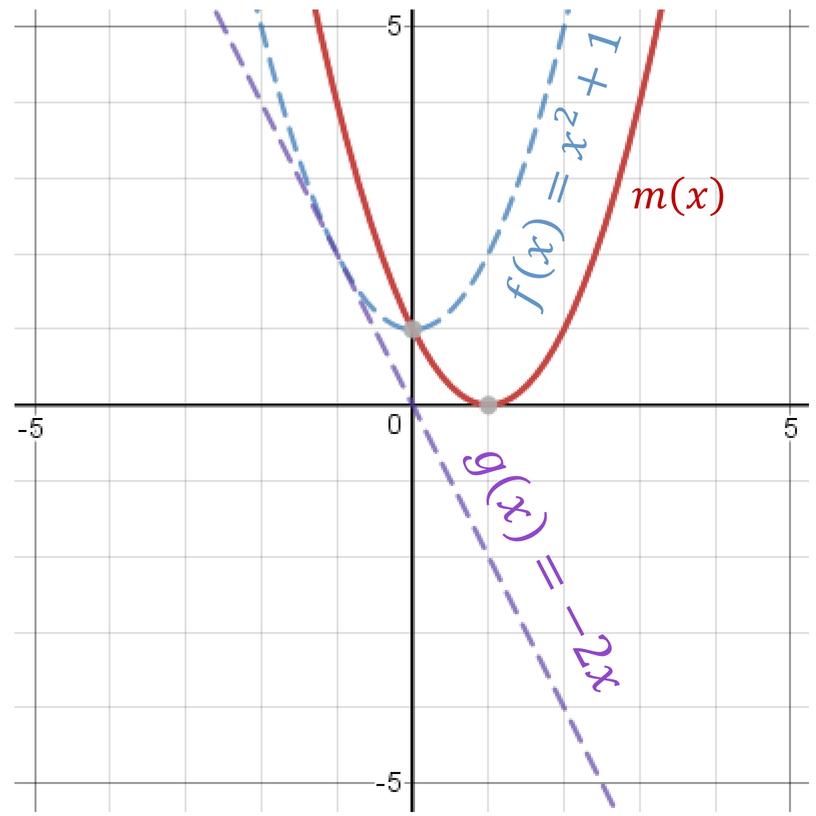
Where the even part is given by:

$$f(x) = \frac{m(x) + m(-x)}{2}$$

And the odd part is given by:

$$g(x) = \frac{m(x) - m(-x)}{2}$$

Example: Even-Odd decomposition of $m(x) = (x-1)^2$



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2.3: McLaurin power Series representation of a function

The **McLaurin** series of a real or complex-valued function $m(x)$ that is infinitely differentiable, is given by the infinite expression (which is a private case of a Taylor series, that is developed around zero):

$$m(x) = m(0) + \frac{m'(o)}{1!}x + \frac{m''(o)}{2!}x^2 + \frac{m^{(3)}(o)}{3!}x^3 + \cdots + \frac{m^{(n)}(o)}{n!}x^n + \cdots$$

Or using the more compact "Sigma" notation:

$$m(x) = \sum_{n=0}^{\infty} a_n x^n$$

With coefficients given by: $\left\{ a_n = \frac{m^{(n)}(o)}{n!} \right\}$

Where:

$\mathbf{m^{(n)}(o)}$ *Denotes the **n**-th derivate of* m *evaluated at the point* $x = 0$
(The derivative of order zero of $m(x)$ *is defined to be the function* $m(x)$ *itself).*

$\boldsymbol{n}!$ *Denotes the factorial of n (0! Is defined to be 1)*

2.4: McLaurin Series private cases for even / odd functions:

It can be shown that the **McLaurin** series of a real valued **even** function, $\boldsymbol{f(x)}$ that is infinitely differentiable, contains only non-trivial **even-order terms** (in other words, the odd order terms are zeroed):

$$f(x)=f(0)+0+\frac{f''(o)}{2!}x^2+0+\frac{f^{(4)}(o)}{4!}x^4+0++\frac{f^{(6)}(o)}{6!}x^6+\cdots$$

Or using the more compact "Sigma" notation:

$$f(x)=\sum_{n=0}^{\infty}a_n x^n$$

With coefficients given by:

$$\left\{a_n=\frac{f^{(n)}(o)}{n!}\right\}$$

While the evenness of $f(x)$ forces odd coefficients to zero

In a similar manner, the McLaurin series of a real valued **odd** function, $\boldsymbol{g(x)}$ that is infinitely differentiable, contains only non-trivial **odd-order terms:**

$$g(x)=0+\frac{g'(o)}{1!}x+0+\frac{g^{(3)}(o)}{3!}x^3+0+\frac{g^{(5)}(o)}{5!}x^5+0+\frac{g^{(7)}(o)}{7!}x^7+\cdots$$

Or using the more compact "Sigma" notation:

$$g(x)=\sum_{n=0}^{\infty}a_n x^n$$

With coefficients given by:

$$\left\{a_n=\frac{g^{(n)}(o)}{n!}\right\}$$

While the oddness of $g(x)$ forces even coefficients to zero

2.5 Small signal approximation

Let's look again at the general McLaurin power series below:

$$m(x) = m(0) + \frac{m\prime(o)}{1!}x + \frac{m\prime\prime(o)}{2!}x^2 + \frac{m^{(3)}(o)}{3!}x^3 + \cdots + \frac{m^{(N)}(o)}{N!}x^N + \cdots$$

For a small "input" argument to the function, $|x| \ll 1$ we get:

$$|x| \gg |x|^2 \gg |x|^3 \gg |x|^4 \gg \cdots$$

In other words, for such "small input signals" (arguments) to the function, the lower the order of the term, the more dominant it is.

2.6: The spectral decomposition of the terms $cos^n(\omega t)$, $sin^n(\omega t)$

For single-tone CW input signals of the form $Acos(\omega t)$ or $Bsin(\omega t)$, it can be shown by the ***Euler*** and ***De Moivre*** theorems that each of the different power-orders terms present in the output can be spectrally decomposed into a sum of first-order sinusoids as follows:

Spectral decomposition: Odd, positive 'N' (1,3,5…)

Both $cos^N(\omega t)$ and $sin^N(\omega t)$ shall include spectral components in all of the following frequencies:

- ***ω:*** Fundamental frequency
- ***3ω:*** 3rd harmonic product
- ***5ω:*** 5th harmonic product
- ⋮
- ***Nω:*** Nth harmonic product (final)

Positive odd 'N' examples:

$$cos^3(\omega t) = 0.75\cos(\omega t) + 0.25\cos(3\omega t)$$
$$sin^3(\omega t) = 0.75\sin(\omega t) - 0.25\sin(3\omega t)$$
$$cos^5(\omega t) = \frac{5}{8}\cos(\omega t) + \frac{5}{16}\cos(3\omega t) + \frac{1}{16}cos(5\omega t)$$
$$sin^5(\omega t) = \frac{5}{8}\sin(\omega t) - \frac{5}{16}\sin(3\omega t) + \frac{1}{16}sin(5\omega t)$$

Spectral decomposition: Even, positive 'N' (2,4,6…)

Both $cos^N(\omega t)$ and $sin^N(\omega t)$ shall include spectral components in all of the following frequencies:

- ***0ω:*** DC response
- ***2ω:*** 2nd harmonic product
- ***4ω:*** 4th harmonic product
- ⋮
- ***Nω:*** Nth harmonic product (final)

Positive even 'N' examples:

$$cos^2(\omega t) = 0.5 + 0.5\,cos(2\omega t)$$
$$sin^2(\omega t) = 0.5 - 0.5\,cos(2\omega t)$$
$$cos^4(\omega t) = \frac{3}{8} + \frac{1}{2}cos(2\omega t) + \frac{1}{8}cos(4\omega t)$$
$$sin^4(\omega t) = \frac{3}{8} - \frac{1}{2}cos(2\omega t) + \frac{1}{8}cos(4\omega t)$$

? Riddle time!

A 100MHz CW signal is passed through a memoryless nonlinear device (a gain compressed amplifier).

What output frequencies will be influenced by the amplifier 5^{th} order nonlinearity?

Nonlinear device characterization, part 3:

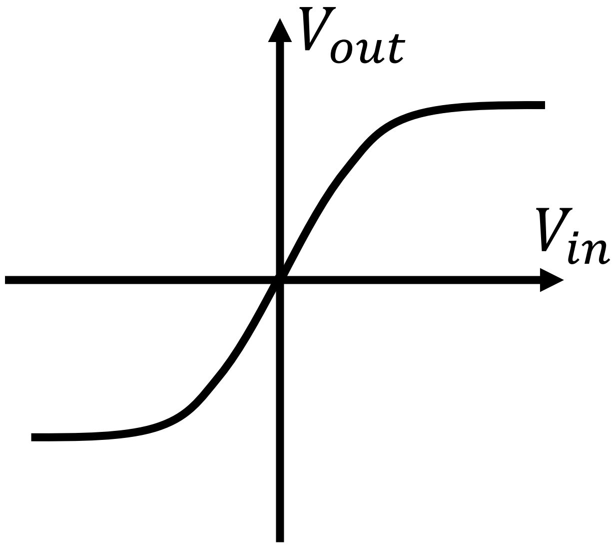
Memoryless RF nonlinearity analysis

The "base model" for memoryless nonlinearity study:
A (loose, yet insightful) **3rd order, odd-function approximation** for the device's actual (infinite series) $V_{out}(V_{in})$ voltage response function:

Let's assume for a moment that our actual DUT (Device under Test, such as an amplifier, for example) has an odd voltage transfer function, which can be described by the following 3rd order approximation (our "Base model"):

$$V_{out}(t) \cong a_1[\,V_{in}(t)\,] + a_3[\,V_{in}(t)\,]^3$$

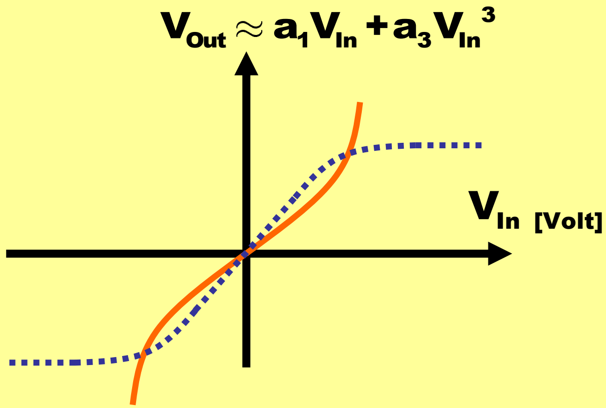
It should be highlighted that the accrual amplifier is "not aware" that we selected to "ease our life" by treating its $V_{out}(V_{in})$ in/out voltage response function as if it was an odd function (which can be in practice very far from truth) and in addition by a 3rd order approximation only.



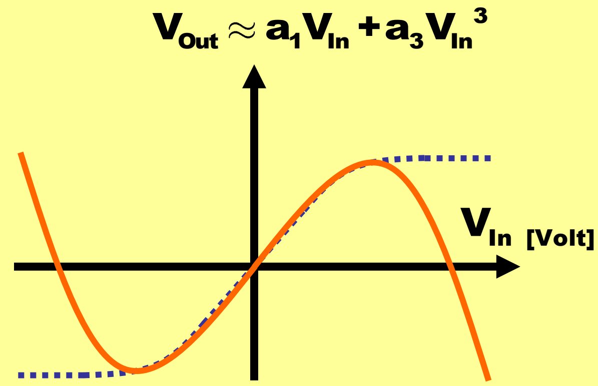
Close to real-life transfer function

The opposite polarities of a_3 and a_1:

The “Base model” transfer function for both a_1 and a_3 positive

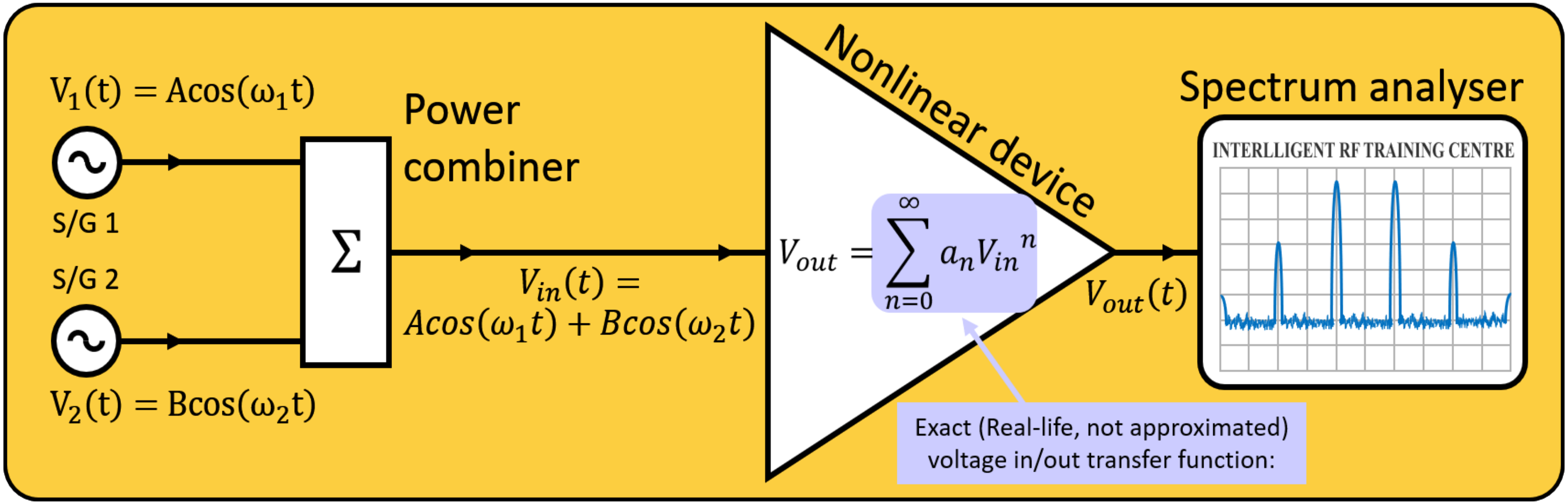


The “Base model” transfer function for a_1 positive and a_3 negative

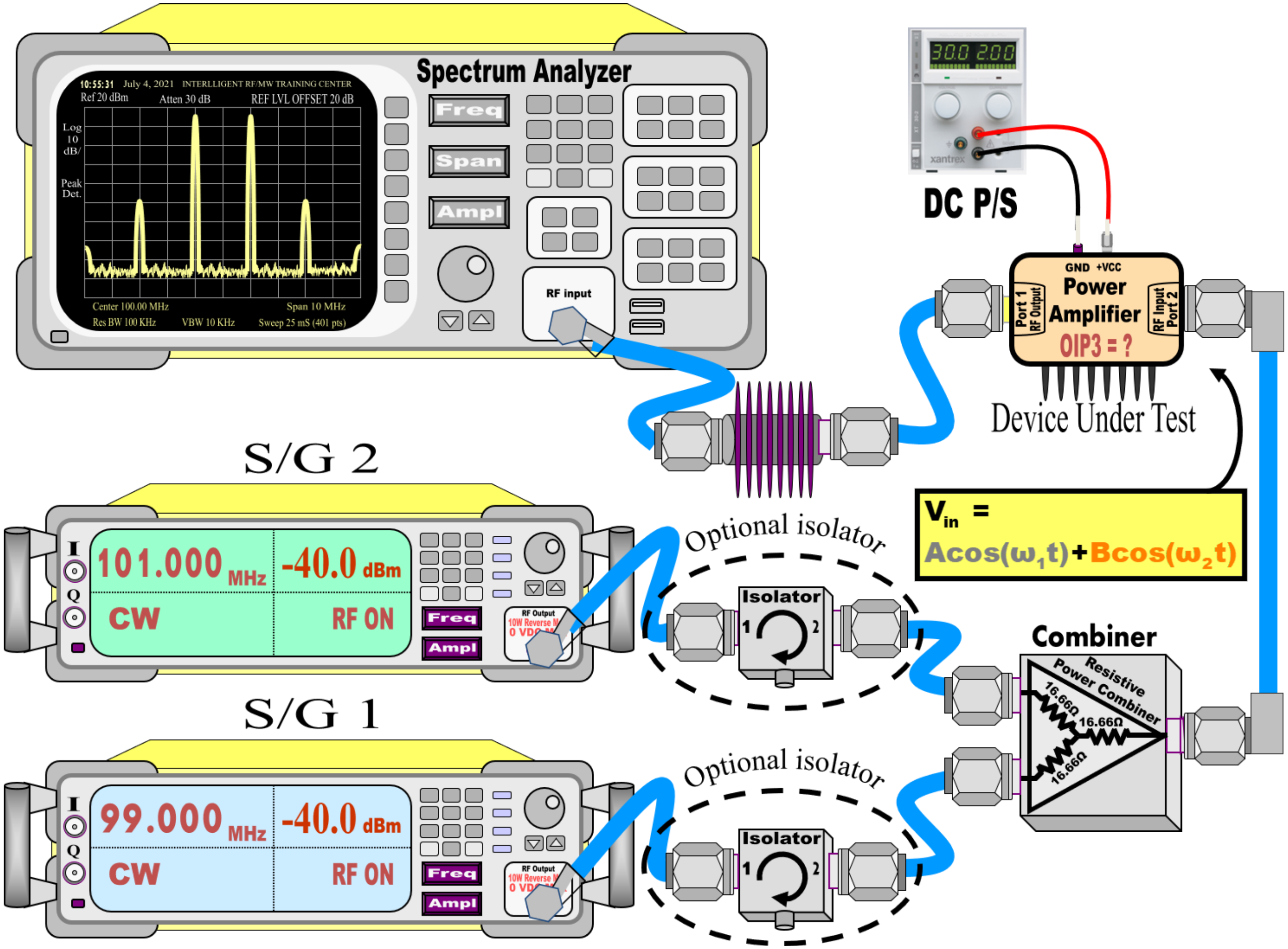


3.2: Two-tone analysis

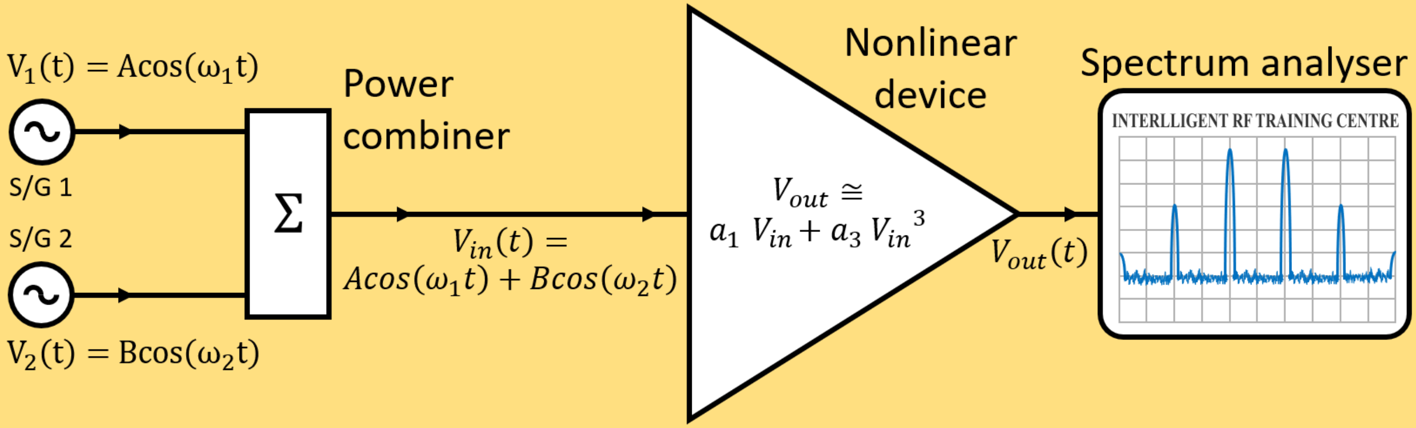
The following 2-tone nonlinearity measurement setup, consists of 2 CW signal generators with their output combined, a nonlinear device under test (the DUT is represented by an amplifier) and a spectrum analyser to measure the device's output:



Practical setup:



Output spectral analysis for a 2-tone input:



Our 3rd order odd nonlinear model: $V_{out}(t) \cong a_1[\,V_{in}(t)\,] + a_3[\,V_{in}(t)\,]^3$

By substituting the combined 2-tone input for V_{in}, we get:

$$V_{out}(t) \cong$$

$$a_1[A\cos(\omega_1 t) + B\cos(\omega_2 t)] + a_3[A\cos(\omega_1 t) + B\cos(\omega_2 t)]^3$$

Investigating the spectral content of the nonlinear device's output, for a 2-tone input:

$$V_{out}(t) \cong \underbrace{a_1[Acos(\omega_1 t) + Bcos(\omega_2 t)]}_{V_{out\,1}} + \underbrace{a_3[Acos(\omega_1 t) + Bcos(\omega_2 t)]^3}_{V_{out\,3}}$$

Linearly amplified "carriers" — 3rd order nonlinear term

Small signal (nominal) gain: By looking at the linearly amplified "carriers" $V_{out\,1}$ term, it becomes obvious that the nominal (small signal) voltage gain is $A_v = a_1$, hence (for non transimpedance devices) the small-signal power amplification factor is $A_p = {a_1}^2$.

Nonlinear term: We now need to investigate the spectral content of the 3rd order nonlinear term, $V_{out\,3} = a_3\, {V_{in}}^3$.

Investigating the spectral content of the nonlinear device's output, for a 2-tone input:

$$V_{out}(t) \cong \boldsymbol{V_{out\,1}(t)} + \boldsymbol{V_{out\,3}(t)} =$$

$$a_1[A\cos(\omega_1 t) + B\cos(\omega_2 t)] + a_3[A\cos(\omega_1 t) + B\cos(\omega_2 t)]^3$$

Linearly amplified "carriers" — 3rd order nonlinear term

Investigating the 3rd order nonlinear term: We will now investigate the spectral content of $\boldsymbol{V_{out\,3}} = a_3 V_{in}{}^3$, or by substituting the 2 tones for V_{in}:

$$\boldsymbol{V_{out\,3}(t)} = a_3 \cdot [\underbrace{A\cos(\omega_1 t)}_{\text{"X"}} + \underbrace{B\cos(\omega_2 t)}_{\text{"Y"}}]^3$$

To be marked as $(X+Y)^3$

By applying Newton's 3rd order binomial identity, we get:

$$(X+Y)^3 = X^3 + 3X^2Y + 3XY^2 + Y^3$$

- X^3: Independent term
- $3X^2Y$, $3XY^2$: "Cross" terms
- Y^3: Independent term

Let's investigate the spectral content of an independent 3rd order term. For example, "X^3":

$$"X^3" \triangleq A^3 cos^3(\omega_1 t) = A^3\left[\frac{3cos(\omega_1 t)}{4} + \frac{cos(3\omega_1 t)}{4}\right] =$$

Identity:

$$cos^3(\alpha) = \frac{3}{4}cos(\alpha) + \frac{1}{4}cos(3\alpha)$$

$$= \frac{3A^3}{4}cos(\omega_1 t) + \frac{A^3}{4}cos(3\omega_1 t)$$

*The "**Gain Compression**" term, that "Eats away" amplitude at frequency ω_1: Since the "X^3" term is multiplied by the McLaurin coefficient a_3 which is (under our 3rd order model's approximation) of opposite polarity to a_1, this product's voltage has an opposite sign to the first order's linearly-amplified carrier at ω_1*

*The **3rd harmonic** product that oscillates at frequency $3\omega_1$: This product's voltage depends on A^3. Therefore, for every 1dB increase in the power of the input carrier at ω_1, this product increases by 3dB.*

Now, let's investigate the spectral content of a "cross" term. Let's go for " $3X^2Y$ ":

$$"3X^2Y" \triangleq 3A^2B\cos^2(\omega_1 t)\cos(\omega_2 t) = 3A^2B\left[\frac{1}{2}+\frac{1}{2}\cos(2\omega_1 t)\right]\cos(\omega_2 t) =$$

Identity (1):
$$\cos^2(\alpha) = \frac{1}{2}+\frac{1}{2}\cos(2\alpha)$$

$$= \frac{3A^2B}{2}\cos(\omega_2 t) + \frac{3A^2B}{2}\cos(2\omega_1 t)\cos(\omega_2 t)$$

Identity (2):
$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha+\beta)}{2} + \frac{\cos(\alpha-\beta)}{2}$$

$$= \frac{3A^2B}{2}\cos(\omega_2 t) + \frac{3A^2B}{2}\cdot\frac{1}{2}\cos[(2\omega_1+\omega_2)t] + \frac{3A^2B}{2}\cdot\frac{1}{2}\cos[(2\omega_1-\omega_2)t]$$

$$= \frac{3A^2B}{2}\cos(\omega_2 t) + \frac{3A^2B}{4}\cos[(2\omega_1+\omega_2)t] + \frac{3A^2B}{4}\cos[(2\omega_1-\omega_2)t]$$

The "Blocking" term: A^2 influences the amplitude at the "other" frequency ω_2

3rd order "sum" intermodulation (IMD) product, at frequency $2\omega_1+\omega_2$ (out of band, close to CF's 3rd harmonic)

3rd order difference intermodulation product, at frequency $2\omega_1-\omega_2$ (in-band, close to CF)

Results summary:

Therefore, the final overall expression for the device's output voltage, under our 3rd order odd-nonlinear "base model" approximation, for the 2-tone input, is given by:

$$V_{out}(t) \cong V_{out\,1}(t) + V_{out\,3}(t) =$$

$$a_1[A\cos(\omega_1 t) + B\cos(\omega_2 t)] + a_3[A\cos(\omega_1 t) + B\cos(\omega_2 t)]^3 =$$

The linearly (1st prder) amplified carriers:

$$= a_1 A\cos(\omega_1 t) + a_1 B\cos(\omega_2 t) +$$

Linearly amplified ω_1 carrier — *Linearly amplified ω_2 carrier*

The $a_3\,X^3$ term:

$$\frac{3\,a_3 A^3}{4}\cos(\omega_1 t) + \frac{a_3 A^3}{4}\cos(3\omega_1 t) +$$

Gain Compression at ω_1 — *3rd harmonic of ω_1*

The $a_3 3X^2Y$ term:

$$\frac{3\,a_3 A^2 B}{2}\cos(\omega_2 t) + \frac{3\,a_3 A^2 B}{4}\cos[(2\omega_1 + \omega_2)t] + \frac{3\,a_3 A^2 B}{4}\cos[(2\omega_1 - \omega_2)t] +$$

Blocking at ω_2 — *3rd order sum IMD at $2\omega_1 + \omega_2$* — *3rd order difference IMD at $2\omega_1 - \omega_2$*

The $a_3 3Y^2X$ term:

$$\frac{3\,a_3 B^2 A}{2}\cos(\omega_1 t) + \frac{3\,a_3 B^2 A}{4}\cos[(2\omega_2 + \omega_1)t] + \frac{3\,a_3 B^2 A}{4}\cos[(2\omega_2 - \omega_1)t] +$$

Blocking at ω_1 — *3rd order sum IMD at $2\omega_2 + 1$* — *3rd order difference IMD at $2\omega_2 - \omega_1$*

The $a_3\,Y^3$ term:

$$\frac{3\,a_3 B^3}{4}\cos(\omega_2 t) + \frac{a_3 B^3}{4}\cos(3\omega_2 t)$$

Gain Compression at ω_2 — *3rd harmonic of ω_2*

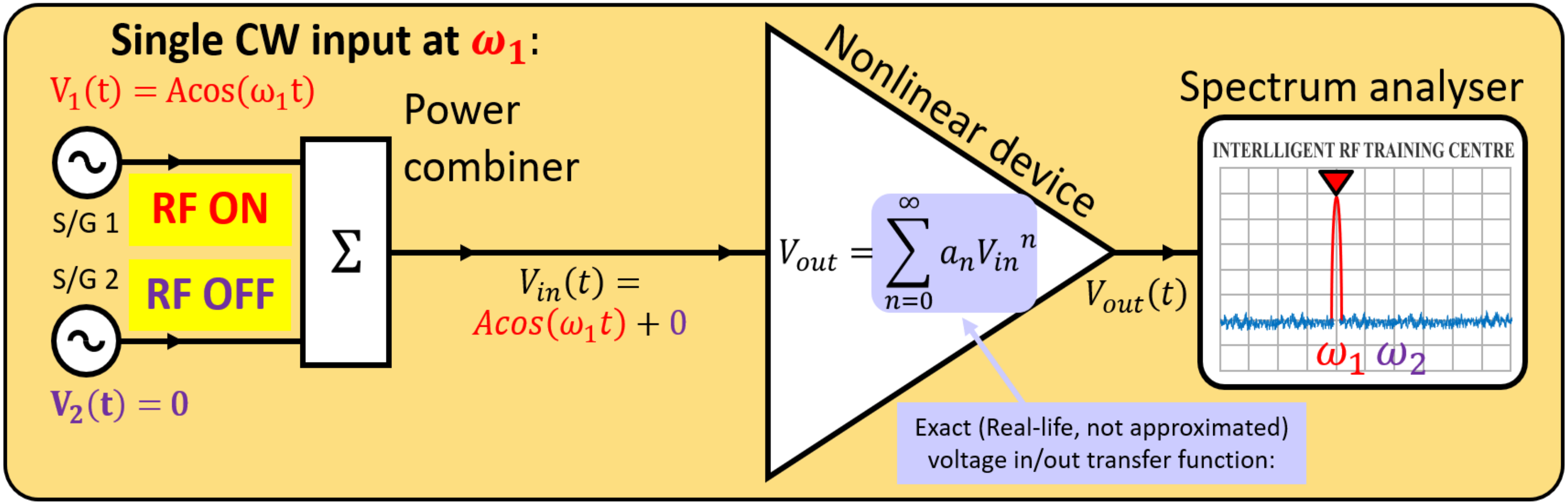
"Blocking" (sometimes also called "De-Sensing") explained:

Let's take a close look on the output amplitude at one of the original carrier's frequencies, for example at ω_1:

Opposite polarities

$$V_{out\ at\ \omega 1}(t) = \left[a_1 A + a_3\left(\frac{3A^3}{4} + \frac{3B^2A}{2}\right)\right]cos(\omega_1 t)$$

Gain Comp. at ω_1 — Blocking at ω_1



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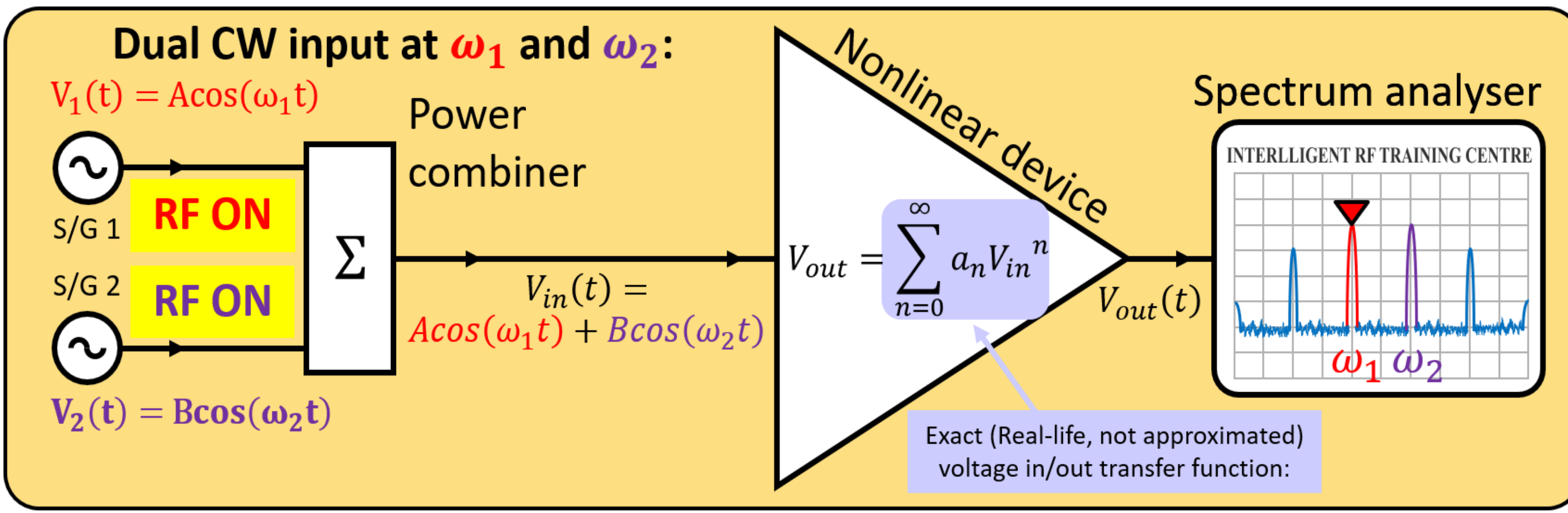
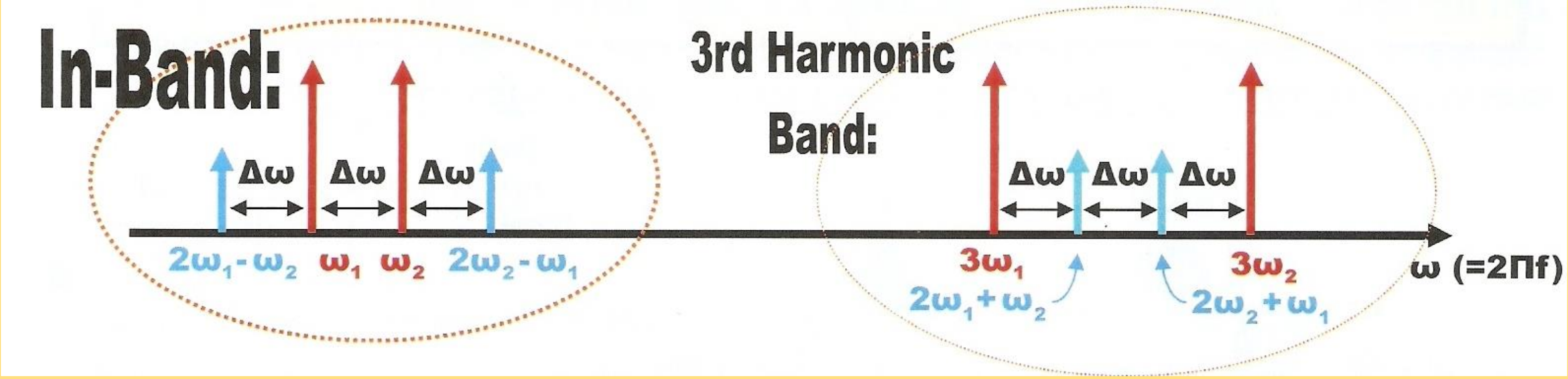
Opposite polarities

$$V_{out\ at\ \omega 1}(t) = \left[a_1 A + a_3\left(\frac{3A^3}{4} + \frac{3B^2A}{2}\right)\right]\cos(\omega_1 t)$$

Gain Comp. at ω_1 — Blocking at ω_1



Results summary in the frequency domain, according to our (non-realistic) 3rd-order odd nonlinear "base model"



? Riddle time!

In case we increase the ω_1 S/G's amplitude by 1dB, and maintain the ω_2 S/G's amplitude, what will be the observed power change at the following frequencies (use small signal approximation)?

- $3\omega_1$
- $2\omega_1 - \omega_2$
- $2\omega_2 - \omega_1$

Results summary, according to our "baseline" (3rd order, odd) model:

#	Output frequency	Exact voltage expression for the output at this frequency, according to the 3rd order odd "baseline mathematical model" selected	Small signal approximation for the voltage expression (most dominant term)	Log-Amplitude dependency (small sig.) $1dB_{in} \rightarrow xdB_{out}$
1	ω_1	$\left[a_1A + a_3\left(\frac{3A^3}{4} + \frac{3B^2A}{2}\right)\right]cos(\omega_1 t)$	$a_1Acos(\omega_1 t)$	$A: 1_{dB} \rightarrow 1_{dB}$
2	ω_2	$\left[a_1B + a_3\left(\frac{3B^3}{4} + \frac{3A^2B}{2}\right)\right]cos(\omega_2 t)$	$a_1Bcos(\omega_2 t)$	$B: 1_{dB} \rightarrow 1_{dB}$
3	$3\omega_1$	$\frac{a_3A^3}{4}cos(3\omega_1 t)$	$\frac{a_3A^3}{4}cos(3\omega_1 t)$	$A: 1_{dB} \rightarrow 3_{dB}$
4	$3\omega_2$	$\frac{a_3B^3}{4}cos(3\omega_2 t)$	$\frac{a_3B^3}{4}cos(3\omega_2 t)$	$B: 1_{dB} \rightarrow 3_{dB}$
5	$2\omega_1 + \omega_2$	$\frac{3\,a_3A^2B}{4}cos[(2\omega_1 + \omega_2)t]$	$\frac{3\,a_3A^2B}{4}cos[(2\omega_1 + \omega_2)t]$	$A: 1_{dB} \rightarrow 2_{dB}$ $B: 1_{dB} \rightarrow 1_{dB}$
6	$2\omega_2 + \omega_1$	$\frac{3\,a_3B^2A}{4}cos[(2\omega_2 + \omega_1)t]$	$\frac{3\,a_3B^2A}{4}cos[(2\omega_2 + \omega_1)t]$	$A: 1_{dB} \rightarrow 1_{dB}$ $B: 1_{dB} \rightarrow 2_{dB}$
7	$2\omega_1 - \omega_2$	$\frac{3\,a_3A^2B}{4}cos[(2\omega_1 - \omega_2)t]$	$\frac{3\,a_3A^2B}{4}cos[(2\omega_1 - \omega_2)t]$	$A: 1_{dB} \rightarrow 2_{dB}$ $B: 1_{dB} \rightarrow 1_{dB}$
8	$2\omega_2 - \omega_1$	$\frac{3\,a_3B^2A}{4}cos[(2\omega_2 - \omega_1)t]$	$\frac{3\,a_3B^2A}{4}cos[(2\omega_2 - \omega_1)t]$	$A: 1_{dB} \rightarrow 1_{dB}$ $B: 1_{dB} \rightarrow 2_{dB}$

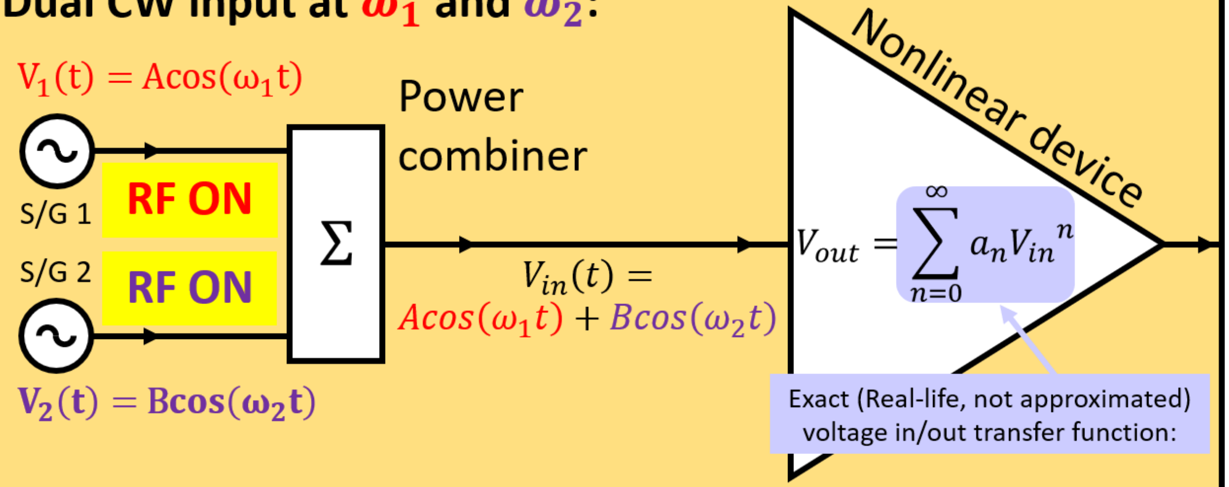
Practical, real-world results for 2-tone nonlinearity analysis:

In real life, the nonlinear device is “not aware” to the fact that in order to gain insight to ins behaviour, we selected an easy-to-analyse, 3rd order, odd mathematical model: In practice, our “baseline” model does not necessarily match the reality.

General case frequency and amplitude results:

(Power results are under the small signal approximation):

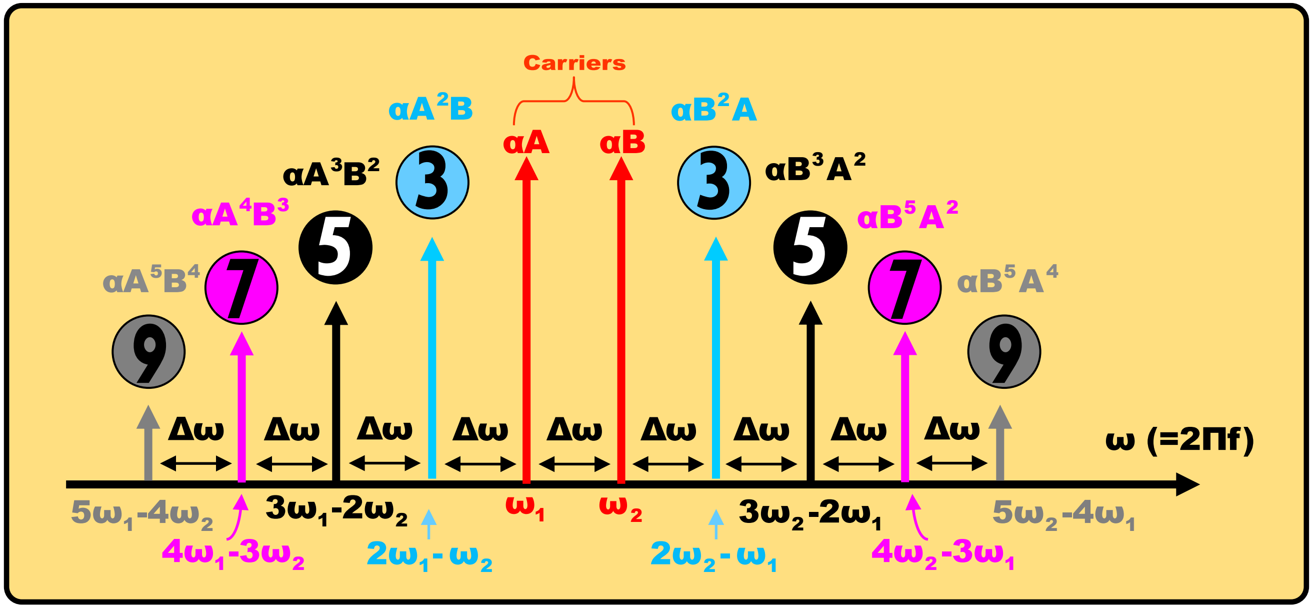
Dual CW input at ω_1 and ω_2:



The output spectrum contains all spectral lines of the form $P\omega_1 + Q\omega_2$, where P and Q are independent (of each other) integers: $P, Q = 0, \pm 1, \pm 2, \pm 3 \ldots$ **The amplitude** of a spectral line at frequency $P\omega_1 + Q\omega_2$ is proportional to $A^{|P|}B^{|Q|}$, and the dominant order of the spectral line is $N = |P| + |Q|$

Practical, real-world results for 2-tone nonlinearity analysis

Practical in-band spectrum (example up to 9th order):



Practical, real-world results for 2-tone nonlinearity analysis

For closely spaced (practically $CF \pm 5\%$) 2 input tones:

Even nonlinear products will emerge in even frequency regions (around even multiples of CF).

While:

Odd nonlinear products will emerge in odd frequency regions Even nonlinear products will emerge in even frequency regions

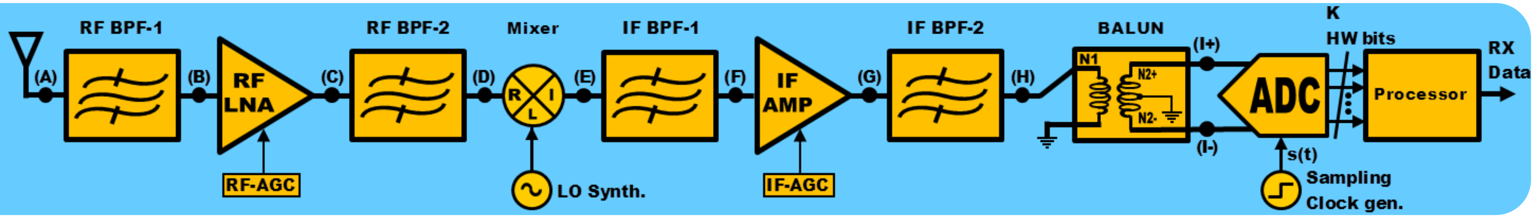
(around odd multiples of CF).

RF and Microwave engineering 101

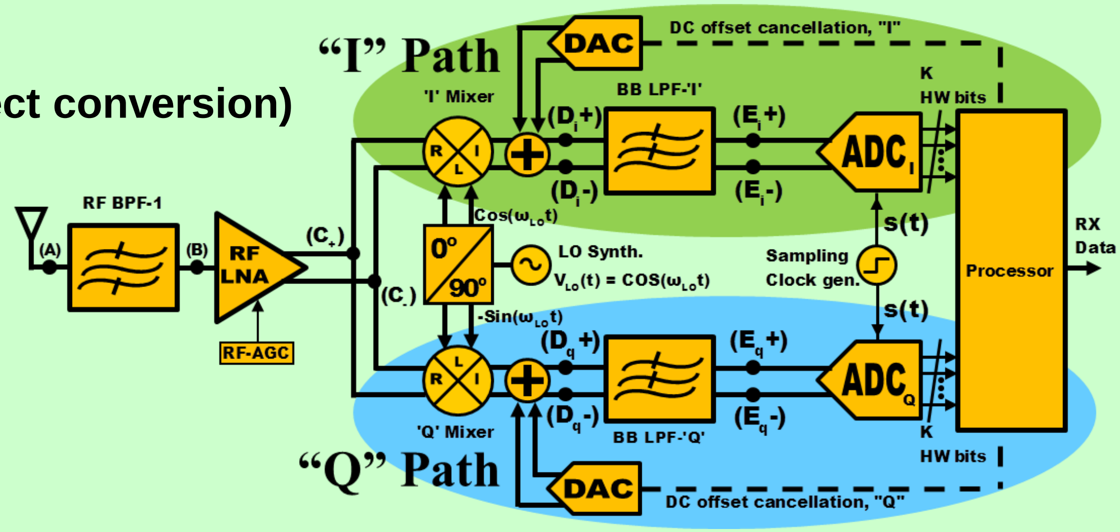
Riddle time!

Which nonlinearity metric (even OIP2 / odd OIP3) is critical for which receiver architecture?

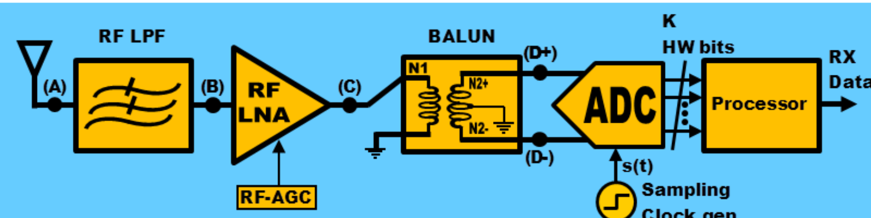
Real-IF (Heterodyne) architecture



Zero-IF
(Homodyne / Direct conversion)



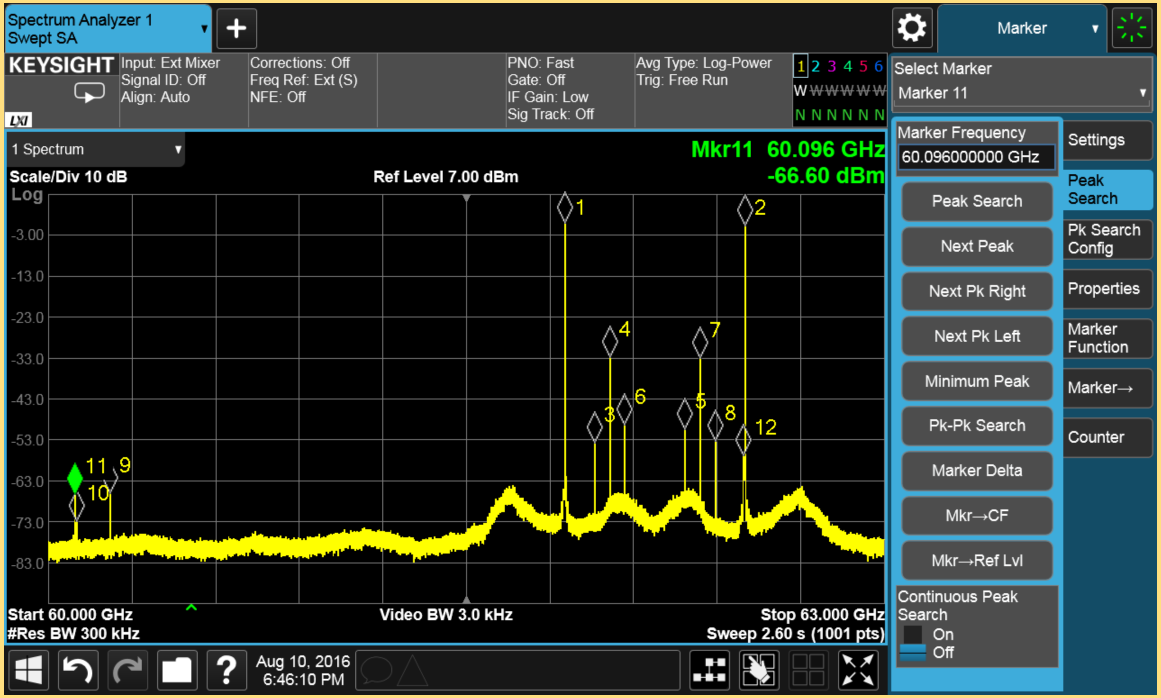
SDR



  Instructor: Oren Hagai

Millimetre wave harmonic mixer - spur identification – LIVE Video example:

Real life sweep. A single 62.5GHz input is fed to a harmonic mixer.



And that's only between 60 to 63 GHz…

Riddle time!

With respect to the 2-tone IMD measurement setup,
The user increased the power of **BOTH** signal generators (SG1 as well as SG2) by 1dB.

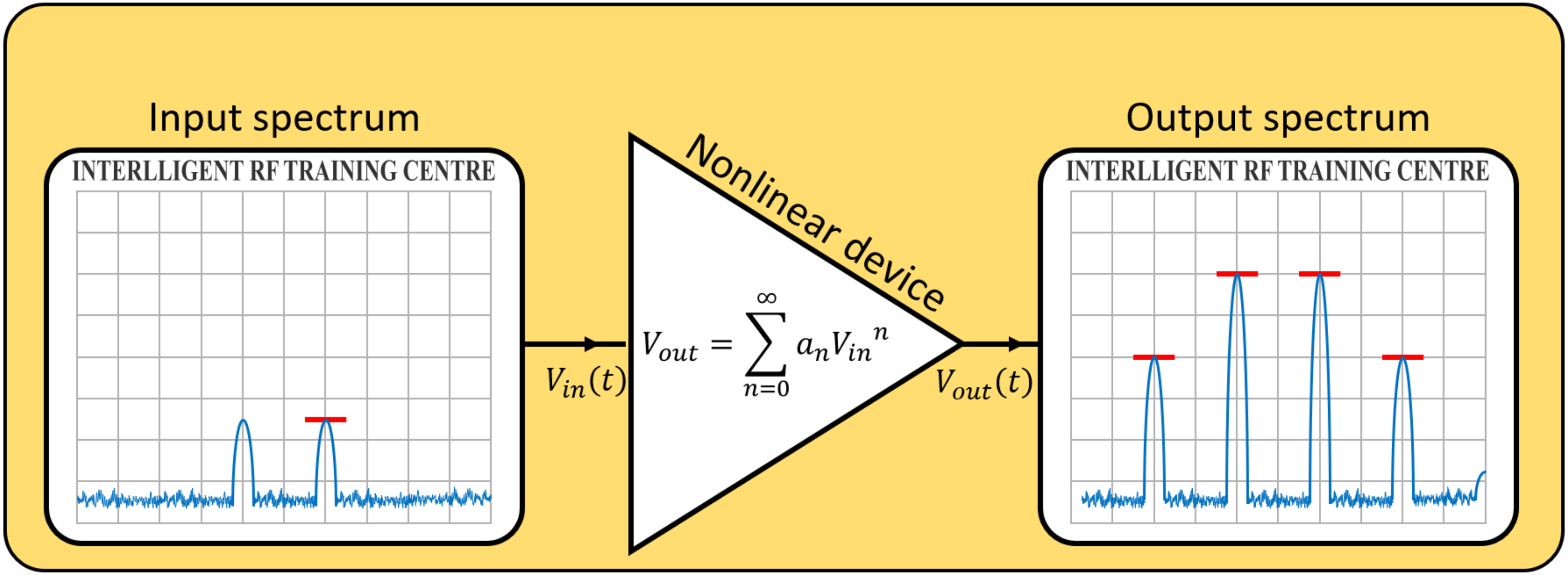
Under the small-signal approximation, what will happen to the power of the 3rd order products at the device's output?

Nonlinear device characterization, part 4:

The interception-point concept

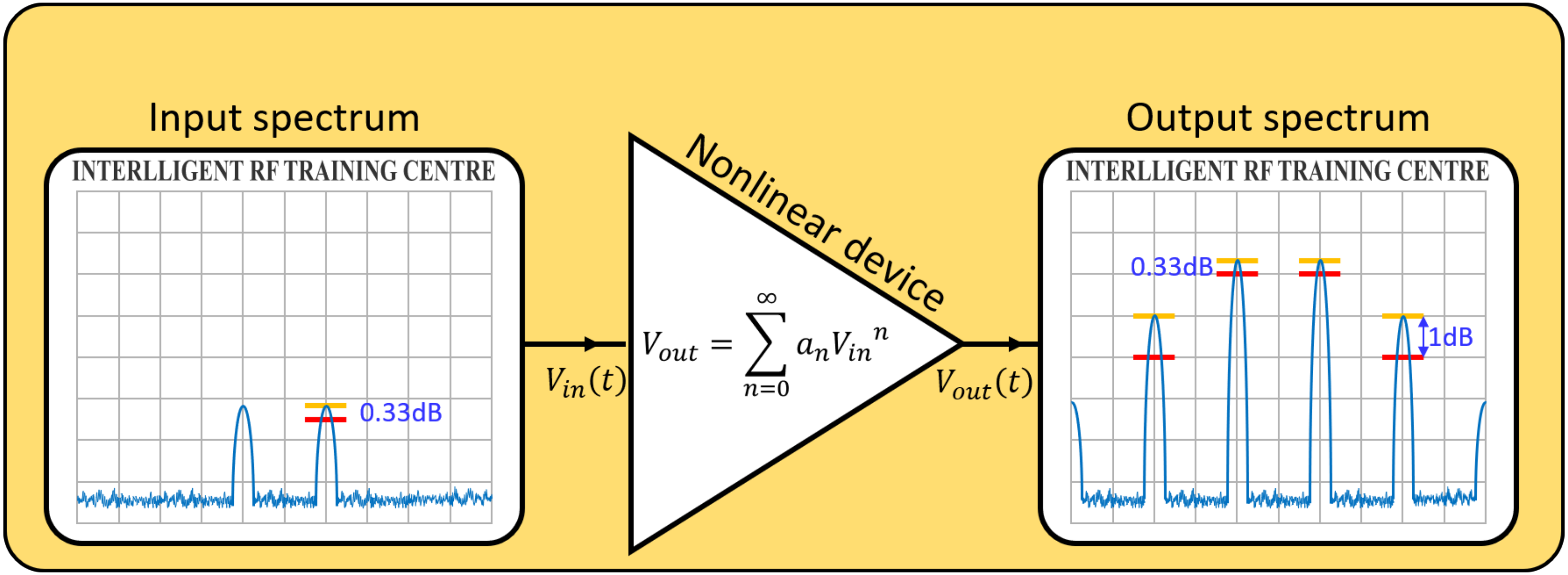
The (theoretical) concept of interception point:

2 equal-power input tones, power sweep, according to our small-signal approximated results:



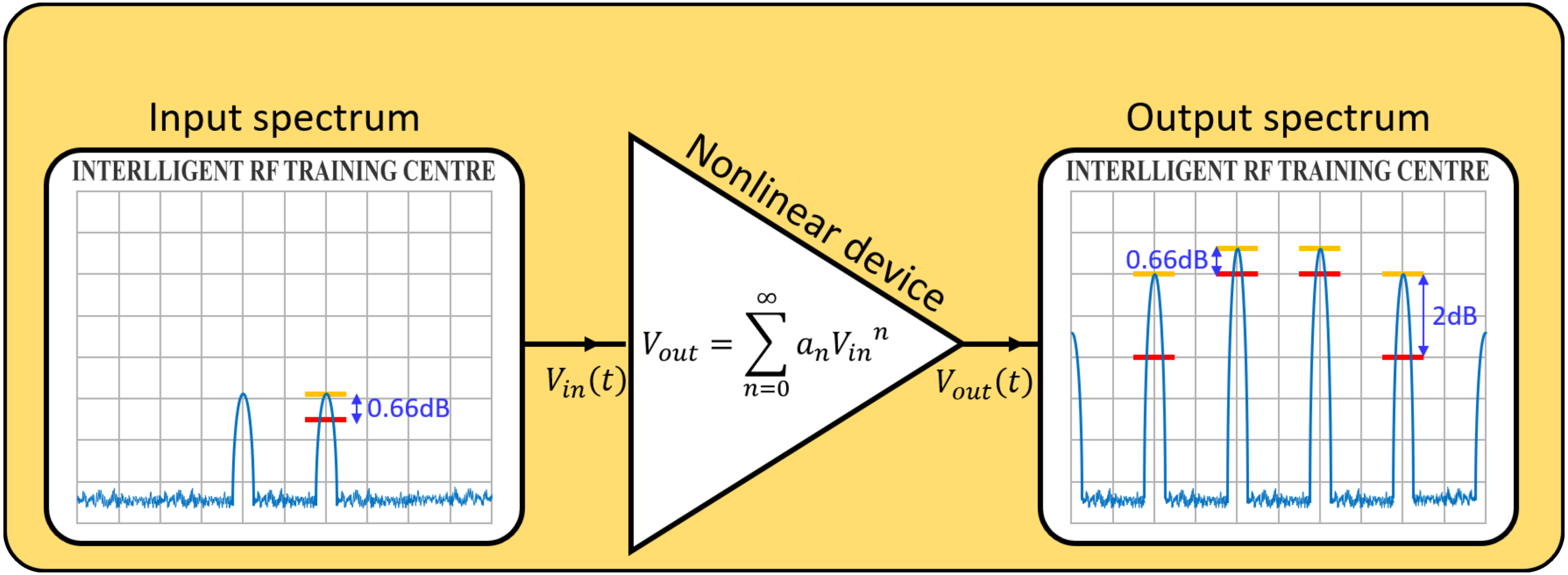
The (theoretical) concept of interception point:

2 equal-power input tones, power sweep, according to our small-signal approximated results:



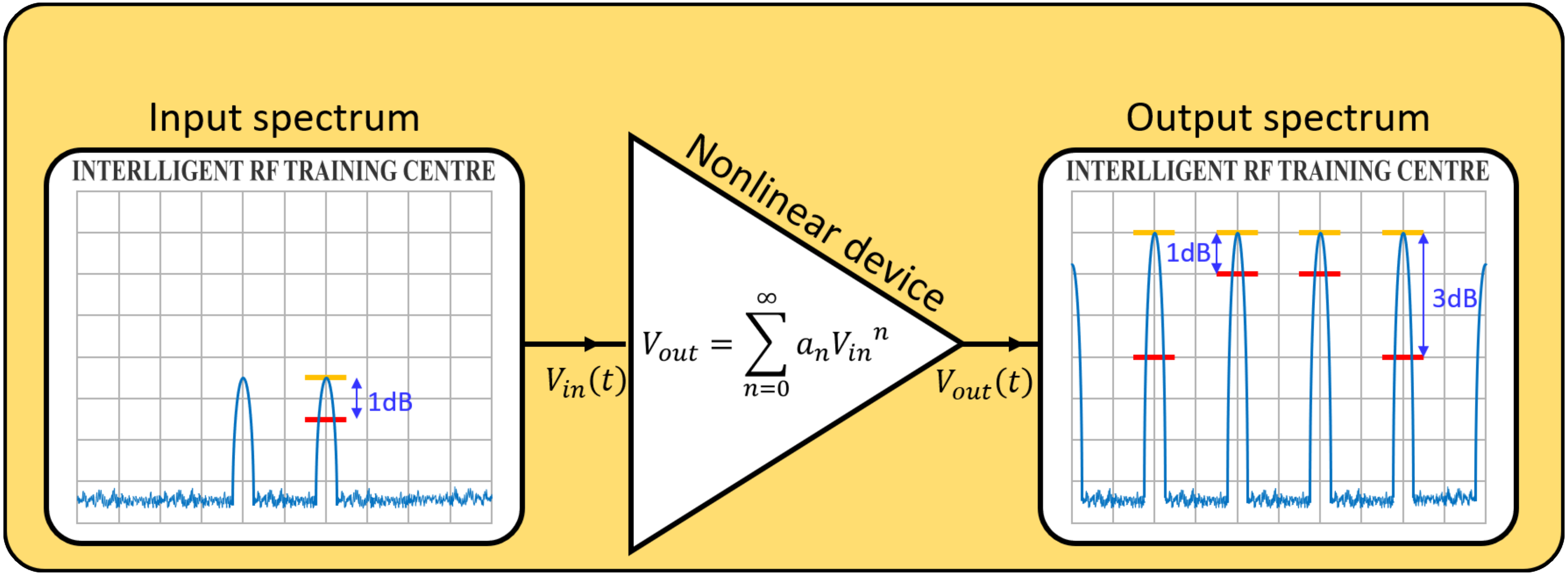
The (theoretical) concept of interception point:

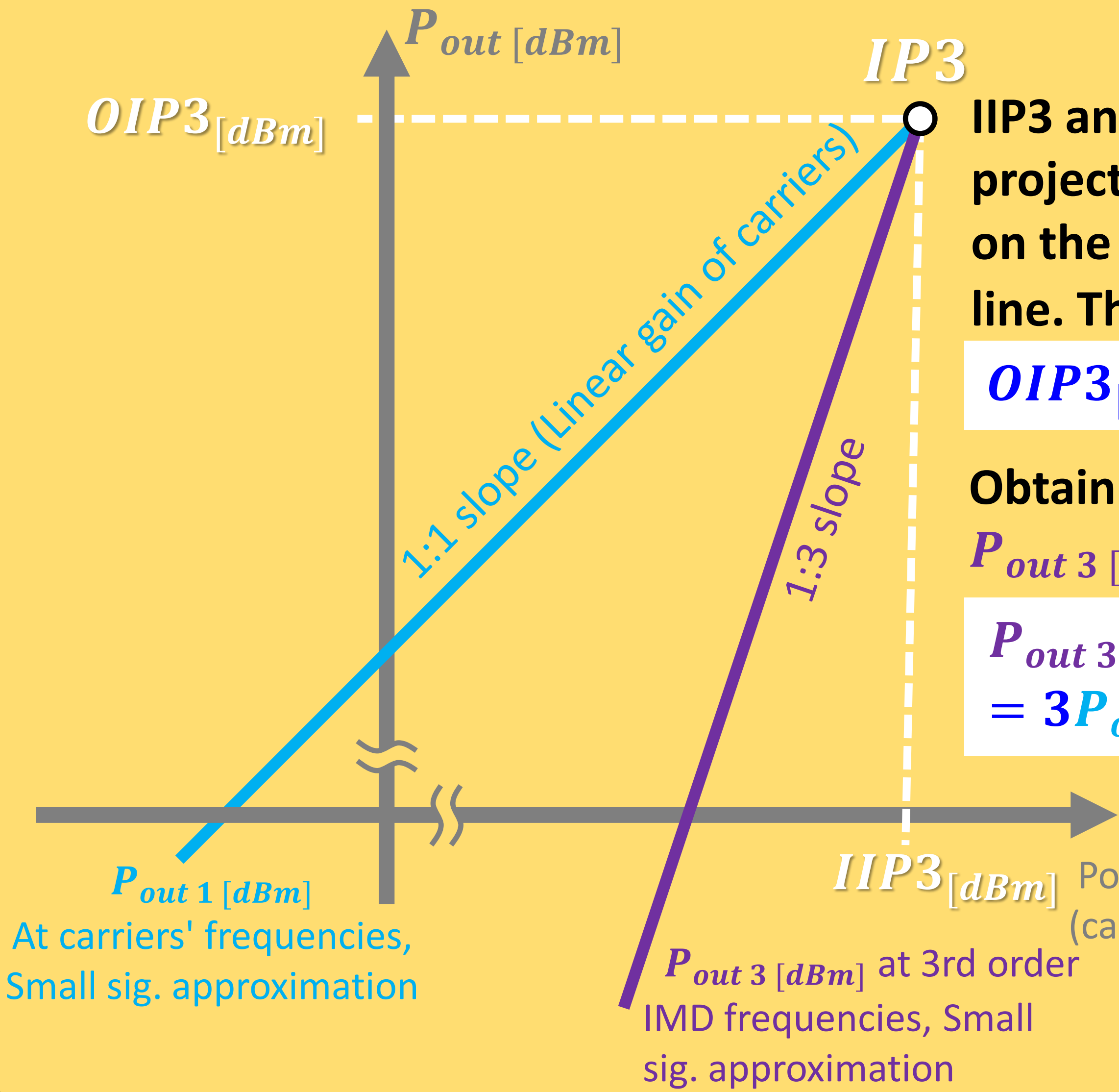
2 equal-power input tones, power sweep, according to our small-signal approximated results:



The (theoretical) concept of interception point:

2 equal-power input tones, power sweep, according to our small-signal approximated results:





IIP3 and OIP3 are the cartesian projections of IP3, which is a point on the $P_{out\ 1\ [dBm]}$ (linear gain) line. Therefore:

$$OIP3_{[dBm]} = IIP3_{[dBm]} + G_{[dB]}$$

Obtaining the line equation for $P_{out\ 3\ [dBm]}$:

$$P_{out\ 3\ [dBm]} = 3P_{out\ 1\ [dBm]} - 2 \cdot OIP3_{[dBm]}$$

Summary of results, IMD's power domain (for equal power carriers, A=B):

"General" N^Th order IMD output power, under the small-signal approximation:

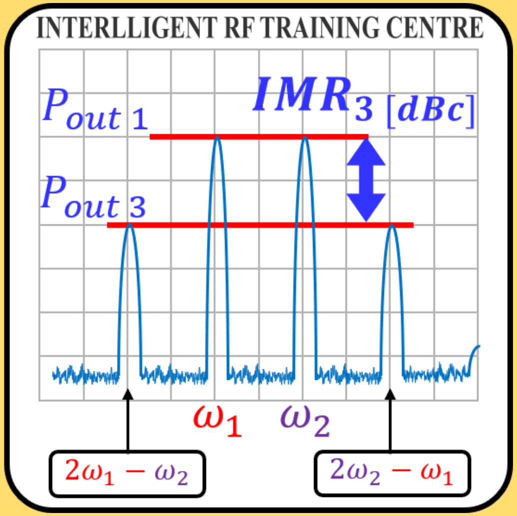
$$P_{out\ Nth\ order\ IMD\ [dBm]} = NP_{out\ 1\ [dBm]} - (N-1)\cdot OIPN_{[dBm]}$$

For intuitive measurements, let's define the "carrier output power to N^Th order IMD power" ratio, IMR-N as:

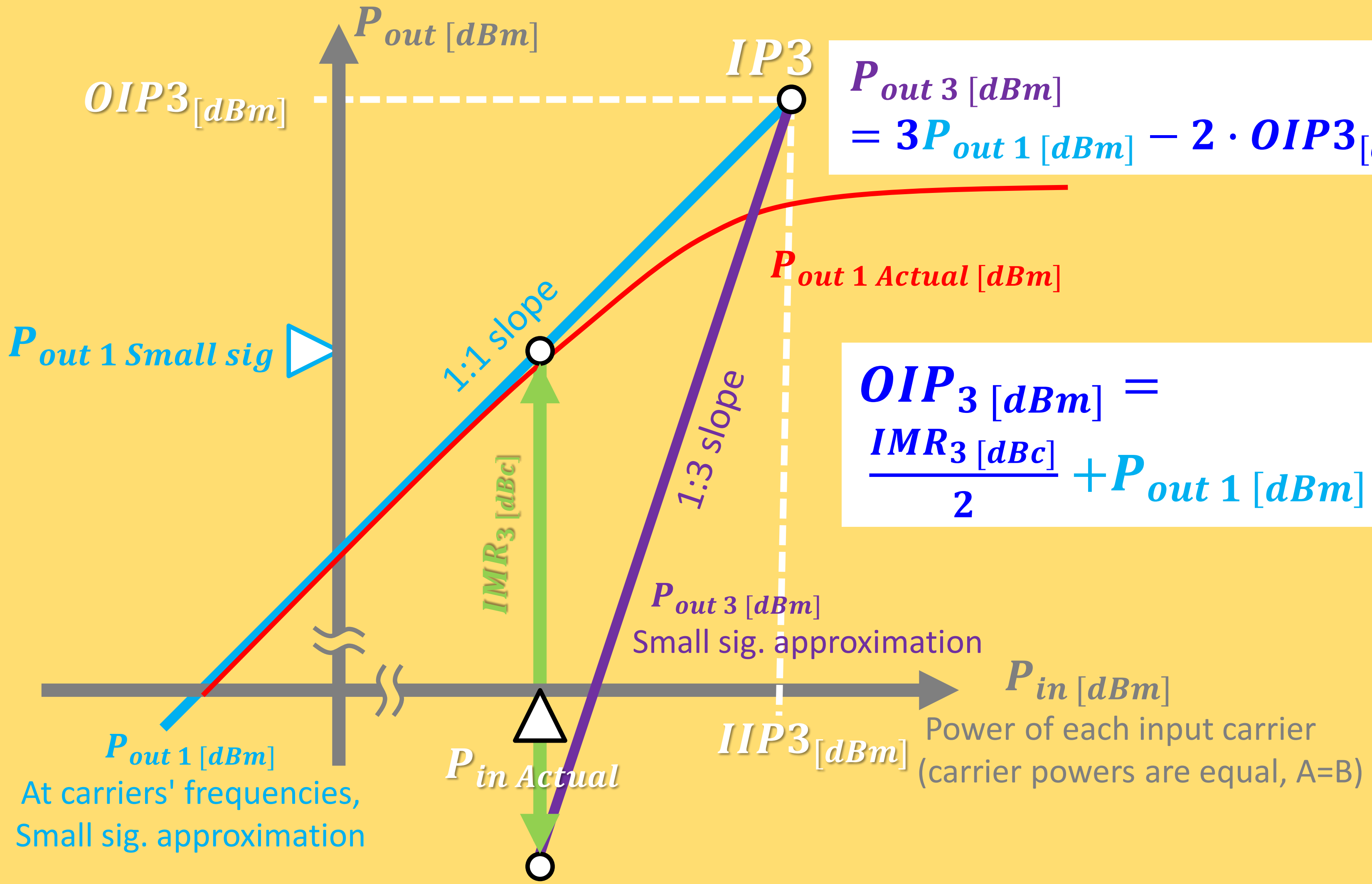
$$IMR_{N\ [dBc]} \triangleq P_{out\ 1\ [dBm]} - P_{out\ Nth\ order\ IMD\ [dBm]}$$

And by substituting the definition of $IMR_{N\ [dBc]}$ in the $P_{out\ Nth\ order\ IMD\ [dBm]}$ line equation, we get:

$$OIP_{N\ [dBm]} = \frac{IMR_{N\ [dBc]}}{(N-1)} + P_{out\ 1\ [dBm]}$$



3rd order nonlinearity, private case analysis:



$$P_{out\ 3\ [dBm]} = 3P_{out\ 1\ [dBm]} - 2 \cdot OIP3_{[dBm]}$$

$$OIP_{3\ [dBm]} = \frac{IMR_{3\ [dBc]}}{2} + P_{out\ 1\ [dBm]}$$

Riddle time!

With respect to the 2-tone IMD measurement setup, The test engineer has increased the power of SG1 (generation the ω_1 carrier) by 1dB. For the removal of doubt, the power of SG2 remained fixed.

In practice, to the engineer's surprise, the measured power at frequency $2\omega_1 - \omega_2$ has increased by 3.2dB (and not by the expected theoretical value of 2dB).

(a) How can this be explained?

(a) What can be modified in the set-up in order to observe the "expected" result of 2dB power increase?

The IP3 model is based on a 3rd order, odd, small signal approximation of the device's $V_{out}(V_{in})$ transfer function.

Practical tip / “trick”

“The OIP3 trick” for simply calculating the overall OIP3 of a nonlinear device followed by a passive lossy (and linear!) device:

Obtaining the overall OIP3 of a chain which consists of a nonlinear device (to be represented as an “amplifier”) followed by a passive, lossy and linear device (to be represented by an “attenuator”) is easy and straightforward:

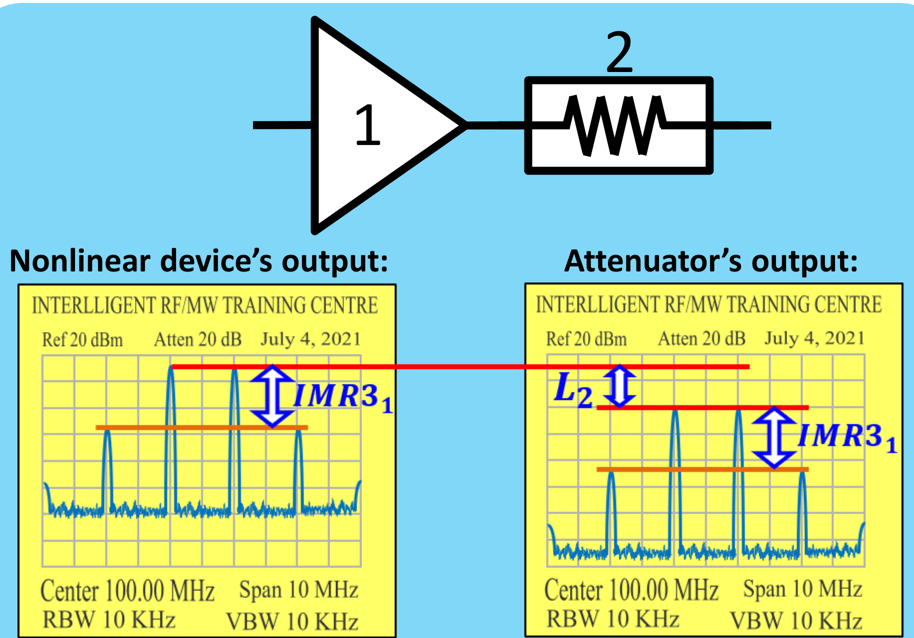
The attenuator “considers” all of its input signals (including the nonlinear products of the nonlinear amplifier) as “first order” signals.

↓

The attenuator linearly attenuates all its input signals by its attenuation L_2 (hence, absolute power decreases while rations, such as IMR remain the same)

↓

According to the OIP3 measurement formula, the overall system’s OIP3 will be the OIP3 of the nonlinear device, minus the attenuator’s loss, L_2:



$$OIP_{3\,[dBm]} = \frac{IMR_{3\,[dBc]}}{2} + P_{out\,1\,[dBm]}$$

→

Result:

$$OIP3_{Tot} = OIP3_1 - L_2$$

OIP3 of an amplifier followed by an attenuator

Nonlinear device characterization, part 5:
Measuring nonlinear products by a spectrum analyser

Instrumentation available for RF power measurements

Spectrum analysers:

- Frequency selective Tuned receiver
- Degraded absolute accuracy (typically 0.1s of dB)
- Widest dynamic range (due to RF step attenuator / Pre-Amp)

Power meters:

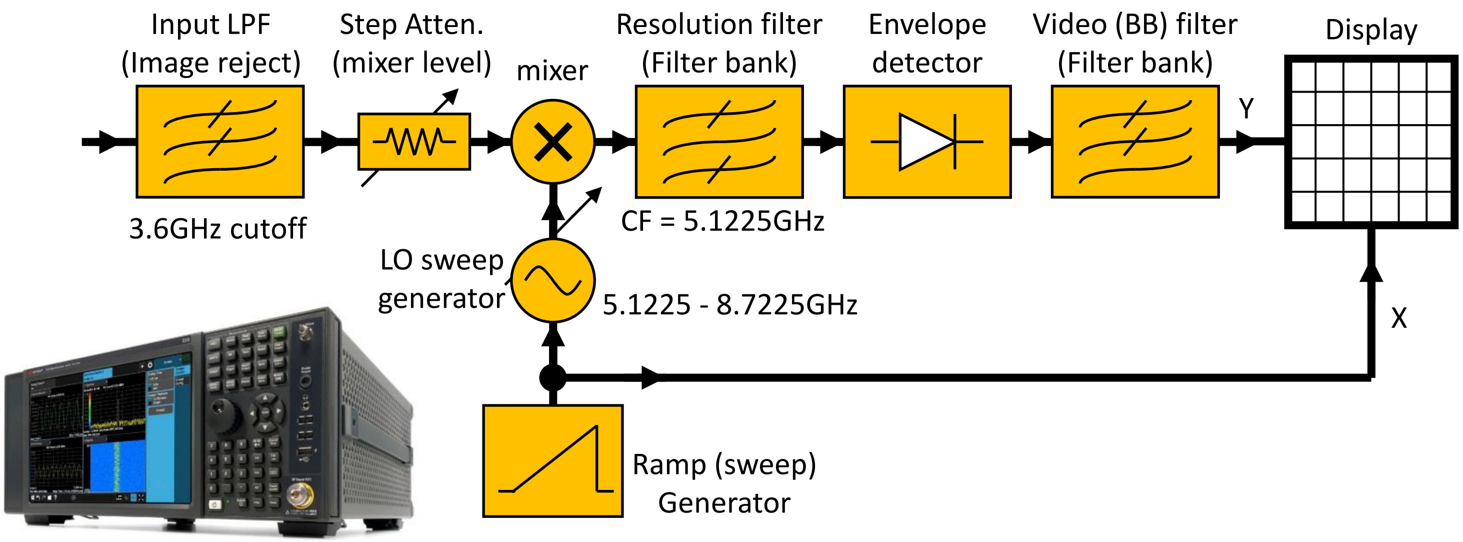
- Non-frequency selective, broadband detection
- Best absolute accuracy (typically 0.01s of dB)
- Smaller dynamic range (due to broadband noise)

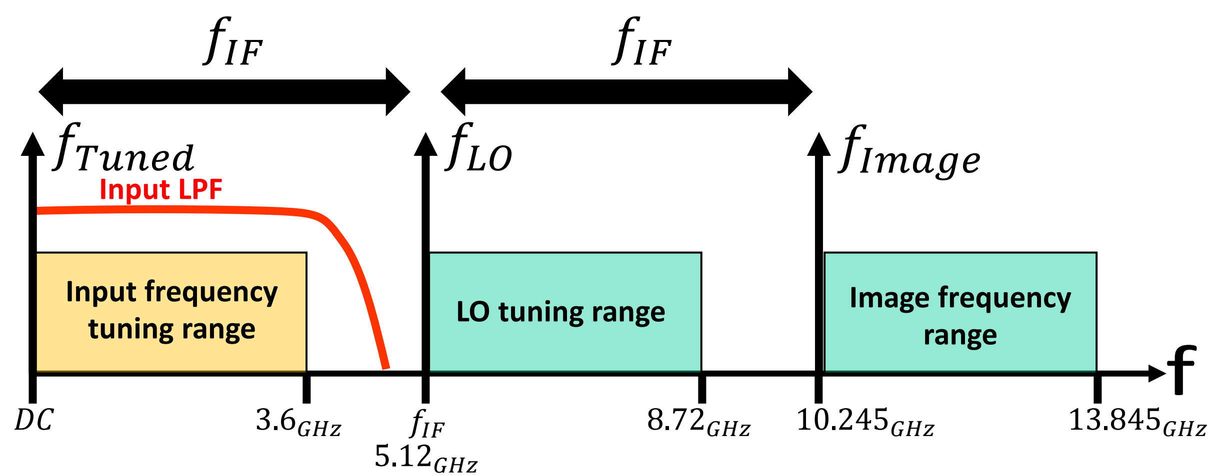
? Riddle time!

Is a Power meter suitable to serve for 1dBcp measurements of an active device?

A simplified block diagram of Frequency swept (Heterodyne) analysers

Example: N9010B-503, 3.6GHz Signal analyser.

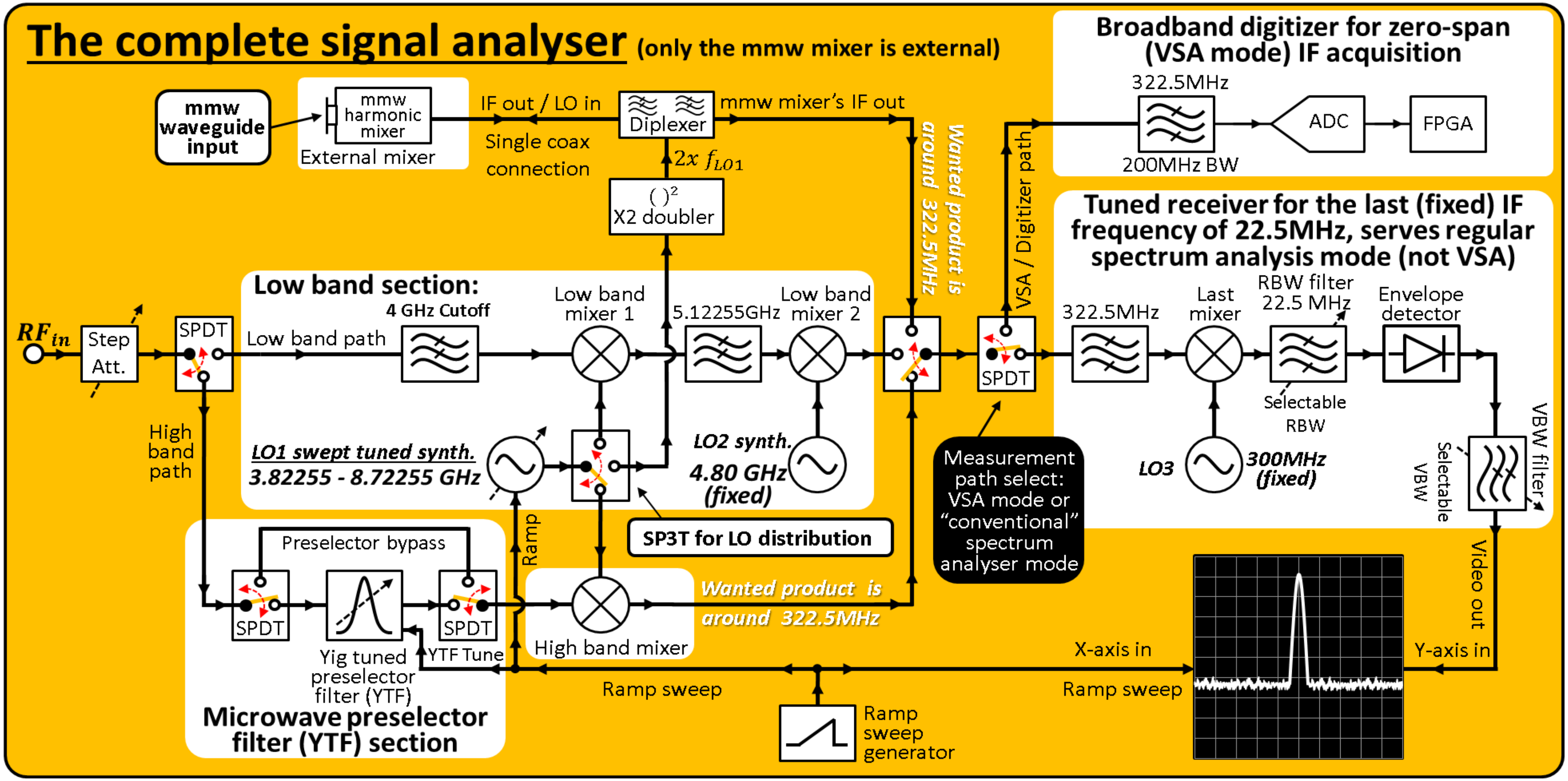




Tuning equation:

$$f_{Tuned} = f_{LO} - f_{IF}$$

A real-life, practical block diagram of a modern signal analyser with external mixing capability:
The block diagram below is based on the structure of modern Keysight "X-series" analysers (N9010/20/30/40B). The signal chain has 3 main paths: **Low-band** (up to 3.6GHz), **High band** (above 3.6 GHz and up to the analyser's highest frequency) and the **external mm-wave mixing** path. In all those paths, the frequency sweep is done only by the 1st LO. The final IF is processed by a tuned receiver (in regular signal analser mode) or by a broadband digitizer (in VSA mode).



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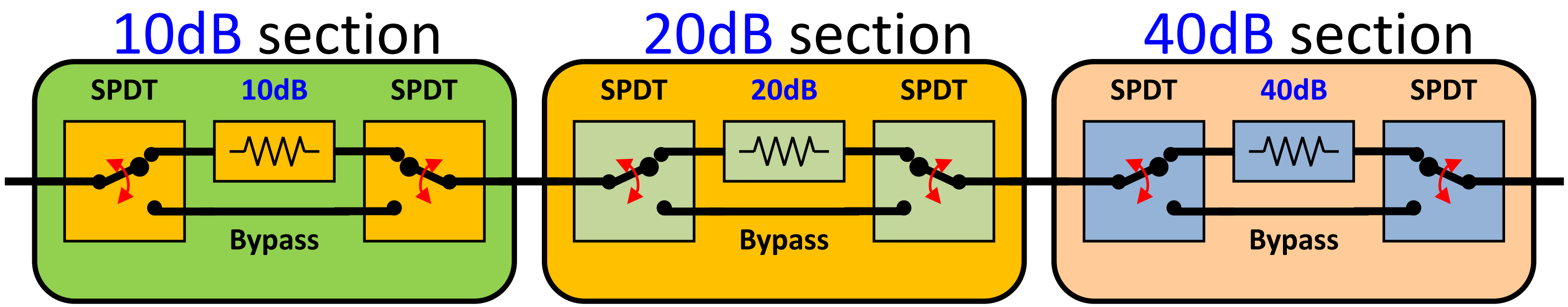
Controlling the S/A's input step-attenuator to assure mixer's linearity:

Usually implemented as a cascaded chain of binary-valued attenuation sections:



Atten.	10dB section	20dB section	40dB section
0dB	OFF	OFF	OFF
10dB	ON	OFF	OFF
20dB	OFF	ON	OFF
30dB	ON	ON	OFF
40dB	OFF	OFF	ON
50dB	ON	OFF	ON
60dB	OFF	ON	ON
70dB	ON	ON	ON

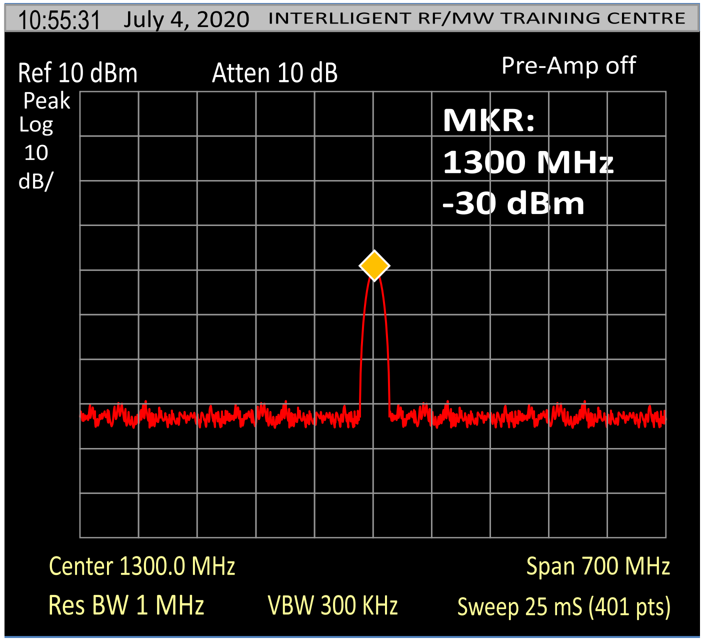
? Riddle time!

<u>Be careful of "hidden" mixer's compression</u>

In un-preselected spectrum analysers, the input mixer is "open to see" all spectral components (from the input) that pass through the input's LPF.

Hence, "unseen" (outside of shown trace) strong signals or even broadband noise may cause mixer compression.
In particular, the RBW filter cannot remove unwanted signals from the first mixer!

Can this 3.6GHz signal analyser actually be gain-compressed?



Verification procedure for "linear fidelity" of signal analysers (which do *not* use external mixers):

*** Not suitable for external mixing due to lack of an internal input attenuator ***

Step 1: Set a nonzero input attenuation and measure the displayed ("to be verified") spectrum:
Measure the amplitude of the displayed signals, using a nonzero internal atten. setting (e.g. Att = 10dB).

Step 2: Expose the analyser's mixer to stronger signal levels and check if the mixer is now compressed:
Decrease the input attenuation by a step or two (example: change the attenuator setting from 10dB into 5dB), and measure again the signal's displayed amplitude. It should be noted that the analyser's CPU always corrects the displayed amplitude to take into account the analyser's internal attenuator setting. pay attention to changes in the displayed amplitudes or to the appearance of any new spectral content

Step 3: Decide if to approve step 1's spectrum + amplitude readings, or start-over with increased attenuation:

Has a significant difference (e.g. new spectral content or an amplitude difference of more than 0.1dB) been observed?

No →

No significant difference observed:
The analyser hasn't shown yet any apparent nonlinear distortion of its own. While the mixer may (or may not) just be now "on the edge" of compression, by restoring the increased input attenuation setting of step 1, we can assure our measurements will practically not be effected by the analyser's nonlinearly. **Hence we can restore the settings of step 1 and rest assured that our spectral content and amplitude readings are indeed valid.**

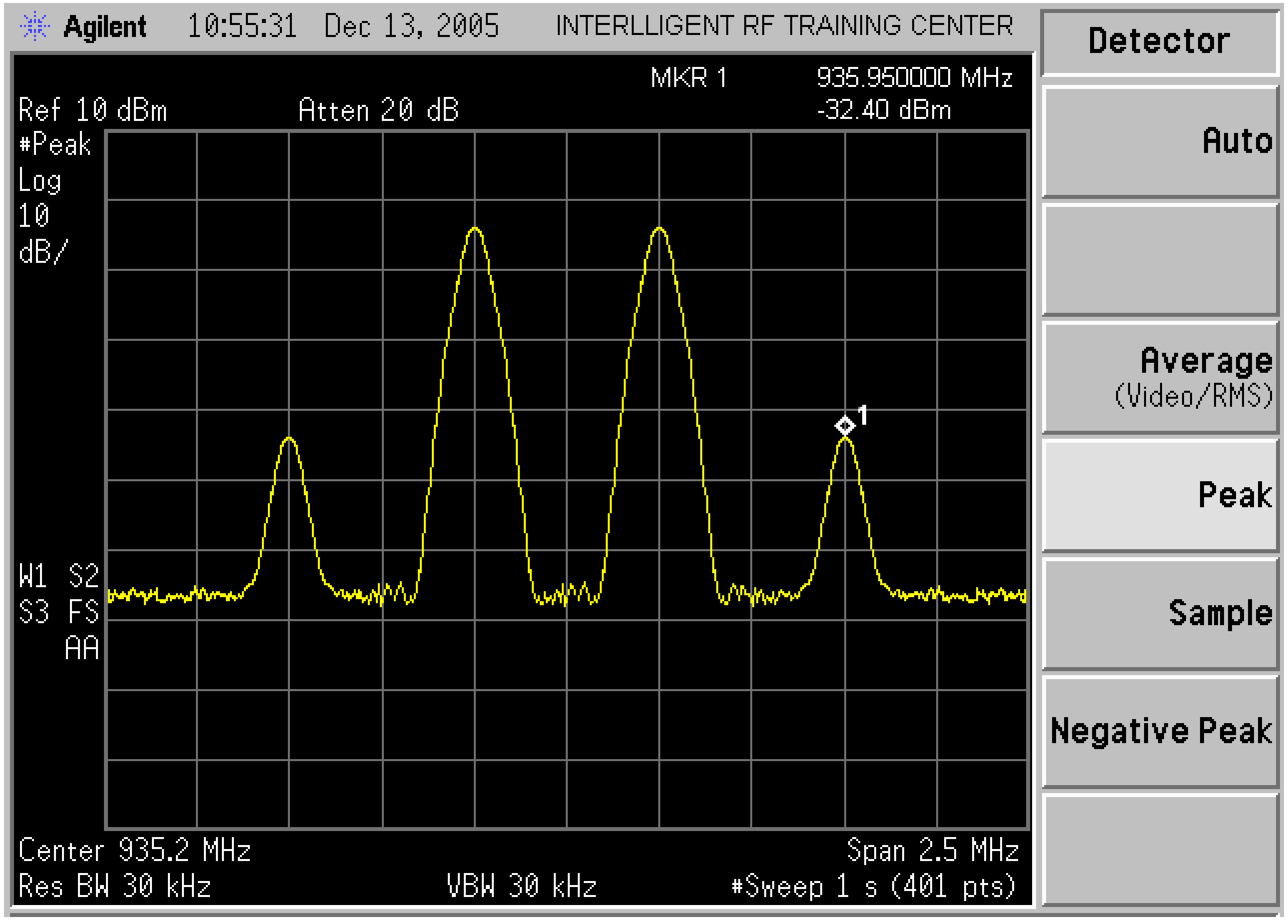
Step 3A:
Perform the measurement using the original (increased) attenuator setting.

Yes →

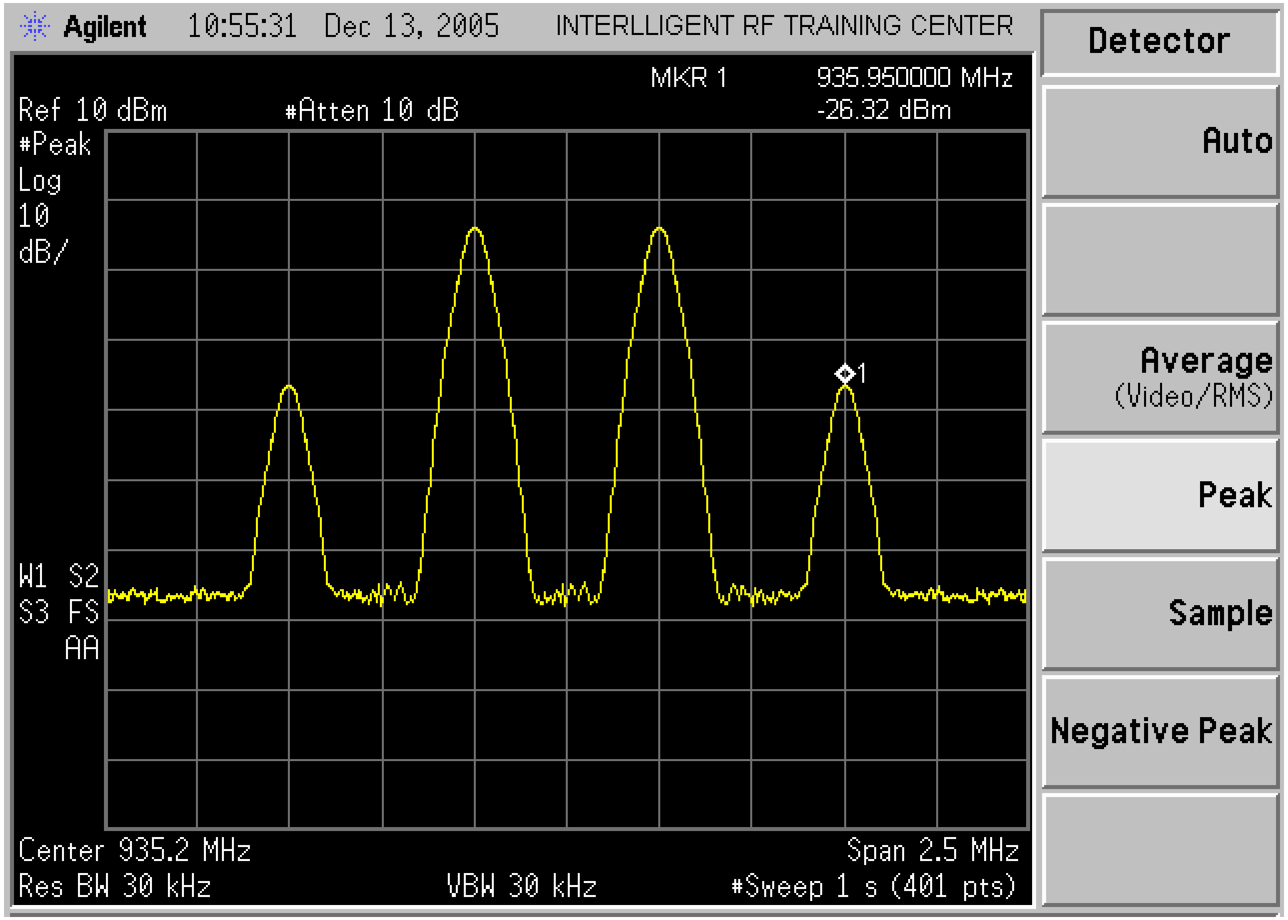
A significant difference has been observed:
It is clear that the analyser's mixer is now gain-compressed. This is due to the fact that the displayed power of the same signals input has changed, or new spectral content has appeared while exposing the mixer to a higher input power. We also cannot be sure this has not been the case when the increased attenuation setting of step 1 was used. **Hence, the readings of step 1 cannot be trusted. We need to start-over with an increased input attenuation.**

Step 3B:
Return to step 1, this time with a higher "starting point" input attenuation.

OIP3 measurement example, step 1:



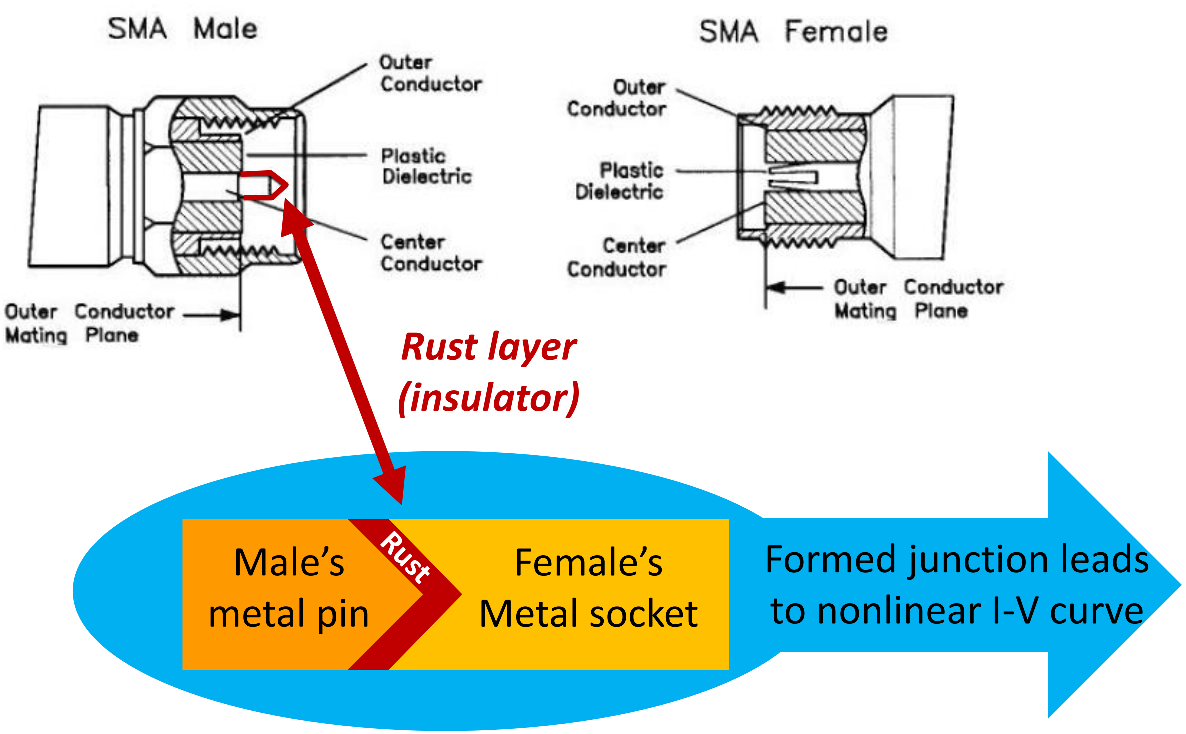
OIP3 measurement example, step 2:

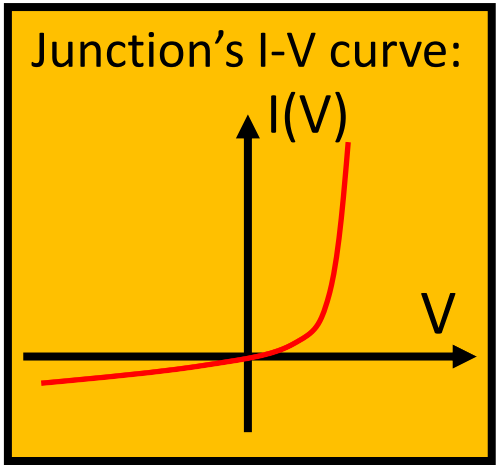


Point of interest

Passive IMD (PIM) caused by corrosion

Rusty or dirty device contacts, connectors or adapters can significantly deteriorate the RF performance of transmitted signal's purity by introducing nonlinear behavior: Corrosion or dirt between 2 metal contacts forms a quantum potential barrier (Fermi gap) that acts as a junction and creates a diode-like exponential I-V curve.





Any questions?

That's it for today!

Don't forget today's homework!

Thank you for attending and see you next week!

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Cell: +972-54-8138200